

A theoretical model of government research and growth

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Abstract

As an institution, the public sector is a major part of a nation's research system. This paper studies the effects of government research on long run growth, assuming that government research is not directly essential to market production but indirectly through its effect on private sector knowledge accumulation. The analysis features knowledge spillovers not only between public and private research sectors but also between research sectors of different countries. The findings: (i) characterize the efficient (growth-maximizing) size of government research; (ii) show that the efficient size is smaller in more open economies; and (iii) indicate that efficient government behavior precludes free-riding on foreign research spillovers (so long as domestic and foreign knowledge accumulation are interdependent). © 1998 Elsevier Science B.V.

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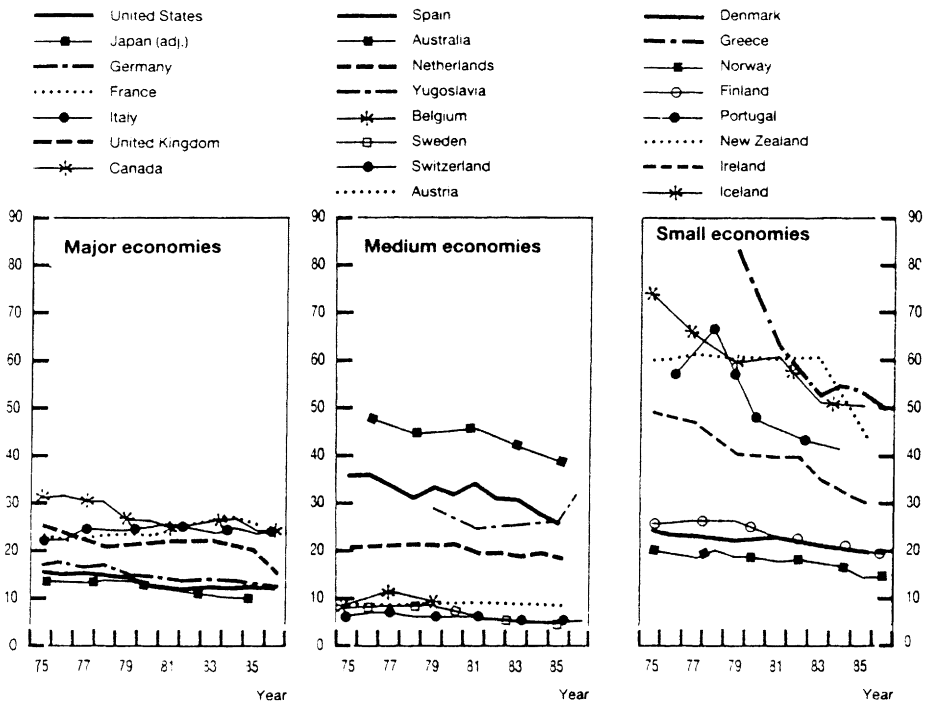
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1. Introduction

Recent growth theories have emphasized the important role of private sector research and development.¹ However, an integral part of a nation's research system is the public sector which not only funds research heavily but actually performs it in various government research establishments. Yet, despite the contribution of government to the accumulation of research knowledge, very little theoretical work exists linking government research to the market economy. This paper helps to fill a gap in the literature by developing a conceptual framework for analyzing the impact of government research on long run growth.

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¹ See, for example, Aghion and Howitt (1992), Grossman and Helpman (1991b), Jones (1995), Romer (1990).



Source: OECD (1989)

Fig. 1. R&D performed in the government sector as percentage of total R&D.

Fig. 1 illustrates the relative importance of the government sector in a nation's research system. There are two key observations: first, the share of research performed by the government varies across countries. The share is generally higher among smaller R&D nations (that is, among nations with smaller R&D to GDP ratios). Secondly, the share of government research has fallen since the mid-1970s. These trends raise a number of issues: what are the consequences for long term growth and innovation? Why has the size of the government research sector declined over time and why is the size smaller for larger R&D nations?

The theoretical analysis addresses these issues by examining what the efficient size and composition of research (between public and private) are in terms of stimulating market production, how government research interacts with private, and how the size of the government research sector varies as the economy becomes more open. A key feature of open economies is the presence of international knowledge spillovers, and the issue is whether governments would free ride on foreign research and thus conduct less R&D.

The previous literature's treatment of government spending falls under two extremes: government spending is either unproductive² or a direct input into private

² See, for example, Frenkel and Razin (1988) and Obstfeld (1989).

production.³ This paper treats the intermediate case where government R&D affects final private production *indirectly*, by working up through microeconomic channels. In a related work, Bartelsman (1991) studies public research and growth in a closed-economy setting and derives results using numerical simulations. This paper extends the analysis to a two-country setting and develops a framework tractable enough to derive results analytically. The analytical approach helps to characterize the underlying properties of efficient government research, and the open-economy setting allows knowledge spillovers to be international in scope (that is, not to be limited geographically). Another related work is Rivera-Batiz and Romer (1991) which studies growth and open-economy spillovers, but focuses on *private* R&D activities. Empirical work on the importance of knowledge spillovers to growth is provided in Coe and Helpman (1995) and Park (1995). This paper can be seen as a theoretical framework for understanding and interpreting the empirical evidence.

The paper is organized as follows: Section 2 describes the closed-economy model, which in turn is the building block for the two-country model that follows. In addition, the closed-economy model can be readily interpreted in the context of the integration of two symmetric countries. Section 3 analyzes balanced growth rates in a two-country world. Section 4 concludes with a discussion of how the developments in Fig. 1 can be interpreted in the context of the theoretical model.

The main points of the paper are as follows, each of which has some empirical significance. First, government research may not contribute directly to market production but indirectly by influencing private research and innovation. If this assumption is correct, it may explain why empirical studies fail to find public R&D to be a significant determinant of private sector productivity⁴ yet find that it is important to private sector research.⁵ Secondly, the long run (reduced-form) relationship between government research and growth is not linear. After the government research sector reaches a certain size, expansions in government research slow growth by crowding out resources from other sectors. Empirically, the effects of government research on growth should therefore depend on the relative size of the government research sector. Finally, the efficient size of the government research sector is smaller in more open economies. Thus, openness is a 'third' factor to consider in the relationship between government research and growth.

2. Closed economy

The model here is an extension of Romer (1990). Romer's basic insight is that growth is caused by the accumulation of a factor (knowledge) that is non-rival but partially excludable. In the extended model, an additional factor that is both non-rival and non-excludable (a pure public good) drives growth.

³ See Shell (1966) and Barro (1990). Barro and Sala-i-Martin (1995) argue that few government expenditures are public goods with the exception of publicly sponsored research and thus interpret the Barro (1990) model to be one of government R&D.

⁴ See for example Griliches and Lichtenberg (1984) and Park (1995).

⁵ See for example Carmichael (1981), Lichtenberg (1984), Mansfield and Switzer (1984), and Leyden and Link (1991).

2.1. Overview of model

The long run growth rate is determined by the interaction of three sectors: the final goods sector, the intermediate goods sector, and the research sector. The research sector consists of both private and government research laboratories. Private sector researchers develop new ideas or ‘designs’ that can be used to produce new durable (intermediate) capital goods. The latter are purchased by intermediate goods producers who then rent them to final goods producers who use the goods as inputs, along with other inputs, to produce a final output. The growth rate of this system depends on the growth of new knowledge (ideas or designs), which in turn depends on the returns to innovation, which depends on the market for new products. Market demand in turn depends on the growth rate of income and consumption opportunities.

Innovation activities are driven by profit-seeking entrepreneurs and inventors who expect to be compensated fully for their efforts. For this reason, when a firm in the intermediate goods sector purchases a design or blueprint from the research sector, it obtains a patent right to be the sole supplier of a durable good based on that design. With free entry-exit and monopolistic competition in the intermediate goods sector, the price of a design (paid by an intermediate goods firm to the research sector) equals the present discounted value of the stream of monopoly profits associated with having a patent right on that design. The monopoly profits come from rental sales to the final goods sector which is the only source of demand for durable goods. The important features about this model are that (i) the design can be commercialized, that is, a capital good can be produced based on its technology and be marketed profitably, and (ii) innovators can appropriate the returns to their innovations.

The model also consists of an infinitely-lived representative agent who consumes the final output, saves, and allocates its human capital among the manufacturing and two research sectors.

The key equations of the model are:

$$U = \int_t^{\infty} e^{-\rho(s-t)} \frac{C(s)^{1-\sigma} - 1}{1-\sigma} ds \quad (1)$$

$$\dot{K} = (1 - \tau)Y - C \quad (2)$$

$$Y = A_p^\alpha H_Y^\alpha K^{1-\alpha} \quad (3)$$

$$Y = rK + wH \quad (4)$$

$$H = H_Y + H_p + H_g \quad (5)$$

$$G = \tau Y = wH_g \quad 0 < \tau < 1 \quad (6)$$

where Y denotes final output, C consumption, G government spending, τ tax rate, r real interest rate, and H human capital. The subscripts Y, p, and g refer to the manufacturing, private research, and government research sectors, respectively. A_p is the stock of private sector knowledge (or *measure* of patented durable inputs available), and K the stock of physical capital.

Eq. (1) is the representative agent’s welfare functional and Eq. (2) the flow budget constraint. The intertemporal optimizing condition is thus $\dot{C}/C = (r(1 - \tau) - \rho)/\sigma$.

Eq. (3) is the production function, which exhibits constant returns to scale in human and physical capital, but increasing returns in all three inputs. Eq. (4) is the factor distribution of income. The agent's income consists of interest and wage income. The agent earns its wage income from supplying human capital services to the three sectors, as shown by Eq. (5), the human capital resource allocation constraint. Eq. (6) indicates that the government balances its budget continuously. The government employs H_g amount of human capital at the wage rate w , and finances this expenditure from income tax revenues.

2.2. Research sector

Following Bartelsman (1991), it is assumed that the research sector consists of two sub-sectors: a private research sector and a government research sector. Each employs human capital to create 'knowledge,' or research output. The link between the research sector and the aggregate economy is K , the stock of all durable capital goods produced. Each durable good is associated with a blueprint developed and sold by the *private* research sector. Each blueprint at some instant fetches a market price of p_A , and the set of blueprints produced at that instant costs wH_p to make (which is just the cost of employing human capital). These blueprints then add to the stock of private knowledge A_p . Thus, the private research sector chooses H_p to maximize the following:

$$\pi = p_A \dot{A}_p - wH_p \quad (7)$$

where

$$\dot{A}_p = \delta H_p F_p(A_p, A_g) \quad (8a)$$

$$\dot{A}_g = \delta H_g F_g(A_p, A_g) \quad (8b)$$

Eqs. (8a) and (8b) describe how the stocks of private and government research knowledge accumulate. Scientists and engineers in private and government research laboratories (that is, H_p and H_g) conduct research. Their productivity depends on the existing stocks of knowledge. F_p and F_g are the knowledge bases available in the private and government research sectors, respectively. For analytical tractability, F_p , F_g will be assumed to be Cobb–Douglas:

$$F_p = A_p^{\beta_p} A_g^{1-\beta_p} \quad 0 < \beta_p < 1$$

$$F_g = A_g^{\beta_g} A_p^{1-\beta_g} \quad 0 < \beta_g < 1$$

Under this formulation, neither private nor government research knowledge substitutes perfectly for the other.⁶ Note that human capital H and the knowledge base F enter multiplicatively.⁷

⁶ That is, the two research sectors produce different kinds of knowledge. The government sector may create knowledge useful for defense, space, and environmental research; the private sector for industrial, agricultural, construction, and consumer goods. While there may be some overlapping knowledge, for example, mathematical and scientific knowledge, this knowledge may be tailored more for research in a particular sector.

⁷ An analogy might be researchers and libraries. Libraries can be represented by F (containing learning materials) and researchers represented by H . Knowledge cannot be augmented by researchers without libraries nor by libraries without any researchers. Both are needed to augment knowledge.

Since each new *private* research output is commercialized, Eq. (8a) can also be thought of as a patent production function. However, Eq. (7) and Eqs. (8a) and (8b) incorporate the assumption that government research does not directly affect market production, but only indirectly. While A_g adds to the pool of existing knowledge (and aids future private and public research), A_p adds to the pool of existing knowledge *and* gives rise to new marketable products.

The research output from the government sector is assumed not to be commercializable for two reasons. First, the output may not be *readily* applicable in the marketplace, if ever (for example, weapons or space technology). As a CBO (1988) study reports:

“... encouraging commercial innovations has never been a principal objective of federal R&D. Instead Congress has largely directed government efforts towards activities that the private sector has limited incentives to perform.” (p.x)

Secondly, even if it can be marketed, government research output is ordinarily not patentable, for the knowledge created is argued to be in the public domain. In any event, private researchers are not precluded from thinking about and developing commercial applications coming out of government research. Furthermore, technologies developed in government research establishments may aid in private research endeavors.⁸

2.3. *Balanced growth rate and efficient size of government*

In a balanced growth equilibrium, consumption, output, capital, and the knowledge stocks grow at the same rate:

$$g = \dot{C}/C = (r(1 - \tau) - \rho)/\sigma = \dot{Y}/Y = \dot{K}/K = \dot{A}_p/A_p = \dot{A}_g/A_g$$

The marginal product of human capital (or wage) in all sectors is the same and the real interest rate $r = (1 - \alpha)\delta H_Y F_p/A_p$.⁹ Solving the model yields the following balanced growth rate:¹⁰

$$g = \frac{(1 - \tau)(1 - \alpha)\delta H - \rho(A_p/F_p)}{(1 - \tau)(1 - \alpha)(A_p/F_p + A_g/F_g) + \sigma(A_p/F_p)} \quad (9)$$

where (F_p/A_p) is the average (knowledge output) productivity of the stock of private knowledge and (F_g/A_g) the average (knowledge output) productivity of the stock of government knowledge. Eq. (9) corresponds to Romer's (1990) growth rate when $\tau = A_g = 0$ and $A_p = F_p$ (that is, when government research is not essential, i.e. $\beta_p = 1$).

⁸ Examples of technologies that were developed or improved in government research labs and that aid industrial R&D include particle accelerators, oceanographic vessels, satellites, computers (see OECD, 1989, p. 52), and semi-conductors, X-ray lithography, high-resolution systems, and biotechnological discoveries (see CBO, 1988, p. 71).

⁹ The real interest rate is obtained from two conditions: (i) $p_A = \alpha(1 - \alpha)Y/rA_p$, that is, the price of a design (under free entry/exit) equals the representative intermediate goods producer's present discounted value of profits (from renting the durable input to the final goods sector), and (ii) $p_A \delta F_p = \alpha Y/H_Y$, that is, the marginal product of human capital in private research equals that in manufacturing.

¹⁰ The model is solved by first determining the allocation of H (using the fact that the marginal product of human capital is equal across sectors) and then substituting the expression for r into the consumer's intertemporal optimization condition.

Eq. (9) shows that across steady states: (i) Higher research productivity in the laboratories (a higher δ) raises the long run growth rate g ; (ii) Higher patience by savers (a lower ρ) raises g ; (iii) A greater intertemporal elasticity of substitution (a lower σ) raises g . The reason is that the benefits of research arrive later on while the costs are incurred earlier; (iv) An increase in the number of scientists and engineers (a higher H) raises g ; (v) An increase in the average productivity of private knowledge in private knowledge production, F_p/A_p , raises g . Likewise, an increase in F_g/A_g raises g ; (vi) An increase in τ (from $\tau = 0$) raises the long run growth rate but lowers it after τ exceeds some critical rate τ_c .

The last point indicates a non-linear relationship between the long run growth rate g and the size of the government research sector ($\tau = G/Y$). The basis for this is as follows. Define q to be the ratio of the stock of private knowledge to the stock of public, that is, $q = A_p/A_g$. An inverted-U relationship between g and τ exists as long as $q'(\tau) < 0$ and $q''(\tau) > 0$.¹¹ The intuition is that as the tax rate (or size of the government research sector) increases, the ratio of the stock of public knowledge to the stock of private knowledge increases but at a *decreasing* rate (across steady states). This diminishing returns feature plays an important role in causing the balanced growth rate to decrease eventually (as τ is raised further). The reason is that fewer new inventions will be produced by private research laboratories. The production of those inventions requires human capital as well as private and public knowledge capital. While increases in τ increase the stock of public knowledge (which in turn enhances the productivity of private research), they also reduce human capital employment in the private sector (as the public sector hires away scarce human capital). Thus, under diminishing returns, increases in public knowledge capital eventually cannot compensate for the loss in human capital employment in the private research sector. For this reason, the equilibrium growth rate falls as τ increases beyond some critical rate.

Let $\tau_c = \arg \max g(\tau)$, that is, the critical tax rate or ‘efficient’ size of the government research sector (G/Y). From Eq. (9), τ_c solves:

$$\frac{d(F_p/A_p)}{F_p/A_p} = \frac{d\tau}{\tau} \frac{\tau}{1-\tau} (1 + \Lambda_1 \Lambda_2) \quad (10)$$

where

$$\Lambda_1 = \frac{(1-\tau)\tau F_g}{\alpha q A_g} > 0, \quad \Lambda_2 = \frac{d}{d\tau} \left[\frac{A_p}{F_p} + \frac{A_g}{F_g} \right] > 0,$$

$F_p/A_p = q^{\beta_p-1}$, and $F_g/A_g = q^{1-\beta_g}$. The LHS of Eq. (10) is the percentage growth in the *average productivity* of the stock of private knowledge in private research and thus represents the marginal benefits of decreasing q (or increasing the ratio A_g/A_p). The RHS represents a measure of the distortional costs of increasing public research or the marginal costs of decreasing q . The first term on the RHS is the percentage increase in the

¹¹ Detailed derivations are available upon request. A simple way to see that g is concave downward in τ is to recognize that $g = 0$ when τ is either 0 or 1. When $\tau = 0$, no growth in public knowledge occurs; when $\tau = 1$, no growth in private knowledge occurs. As long as $0 < \tau < 1$, the stocks of both public and private knowledge grow at a positive rate, and hence the balanced growth rate, g , is positive.

tax rate and the second term the ratio of the tax rate to the non-tax rate, or an index of distortionary pressure. The third term is a scaling factor.¹²

Eq. (10) can be used (as in the next section) to determine how the efficient size of the government research sector varies as underlying parameters or exogenous variables change. For instance, if the LHS > RHS, it pays to withdraw more private resources in order to expand the government research sector. Compared to the resulting growth in distortions, a unit of private resources withdrawn produces a greater return in terms of a higher growth in the private sector's average research productivity, the outcomes from which will be more marketable innovations, durable goods, and increased productivity in the final goods sector. Conversely if the LHS < RHS, it pays to contract the government research sector and return resources to the private economy.

When the LHS = RHS, $\tau = \tau_c$ and the economy grows at its highest possible steady-state rate. Note that this maximum growth rate need not coincide with the socially optimal growth rate. The focus here, however, is not on welfare issues but on the positive link between public sector research and growth. This emphasis seems appropriate in light of the fact that recent policy concerns are about the sources of growth and about how nations can raise their productivity. For this purpose it is of interest to determine the efficient size of the government research sector at which the growth effects of public R&D reach a peak.

3. Open economy

Consider a two-country world economy. The two countries produce identical final goods and are linked by international lending and borrowing, and knowledge spillovers. In this one (final) good world, the intertemporal terms of trade is r_Z , which is the rate of return on Z , an internationally traded bond. The two countries have identical tastes and technologies in manufacturing and research and the same stock of human capital, H .

The main modifications to the basic model shall first be described. The GDP identities are replaced by current account identities (where asterisks * denote foreign country variables):

$$\dot{Z} = r_Z Z + Y - C - \dot{K} - G \quad (11a)$$

$$\dot{Z}^* = r_Z Z^* + Y^* - C^* - \dot{K}^* - G^* \quad (11b)$$

$$Z + Z^* = 0 \quad (11c)$$

The research sectors are modified as follows:

$$\dot{A}_p = \delta H_p (F_p + \theta^* F_p^*) \quad (12a)$$

¹² $\Lambda_2 > 0$ is optimal (rather than $\Lambda_2 = 0$) since the stock of private knowledge, A_p , contributes not only to knowledge accumulation (as does the stock of government knowledge) but also to market production (which the stock of government knowledge does not do). Thus, it is worth obtaining an "extra" increase in F_p/A_p at the expense of a larger decrease in F_g/A_g . The fact that A_p has this additional contribution appears in the last terms of the numerator and denominator of Eq. (9).

$$\dot{A}_g = \delta H_g(F_g + \theta^* F_g^*) \quad (12b)$$

$$\dot{A}_p^* = \delta H_p^*(F_p^* + \theta F_p) \quad (12c)$$

$$\dot{A}_g^* = \delta H_g^*(F_g^* + \theta F_g) \quad (12d)$$

where $F_i = F_i(A_p, A_g)$ and $F_i^* = F_i^*(A_p^*, A_g^*)$, $i = p, g$ and $0 \leq \theta \leq 1$ and $0 \leq \theta^* \leq 1$.¹³ The F_i 's are again Cobb–Douglas functions.

The θ , θ^* are the ‘spillover’ parameters. θ^* represents the degree to which foreign knowledge substitutes for domestic knowledge in domestic knowledge production and θ the degree to which domestic knowledge substitutes for foreign knowledge in foreign knowledge production. If $\theta = \theta^* = 1$, the stocks of foreign and domestic knowledge are perfect substitutes and international knowledge spillovers are perfect. Provided θ and θ^* are non-zero, scientists and engineers in different countries (H_i and H_i^* , $i = p, g$) can draw upon a pool of global knowledge relevant to their sector of research. Note that domestic knowledge stocks enter the knowledge production functions of only domestic (public and private) research but still affect foreign knowledge production since foreign scientists and engineers have access to the domestic knowledge base through international knowledge spillovers.¹⁴ Finally, complete international patent protection exists and all research is non-redundant.¹⁵

Two cases will be considered: (1) *symmetric* spillovers ($\theta = \theta^*$), in which case the two countries are completely symmetrical; (2) *asymmetric* spillovers ($\theta \neq \theta^*$). In this case, governments in the two countries levy different tax rates ($\tau \neq \tau^*$). Case 1 shows how the growth rate changes in an open economy and shows that the efficient size of the government research sector is smaller in an open economy. Case 2 shows that the country with the larger spillover parameter has the smaller efficient government research sector, and also shows why efficient national governments would not free-ride on international knowledge spillovers.

The implications of these results are that, with greater international knowledge spillovers, public sector research becomes more efficient and can be economized (hence the decrease in *size*). The reduction in public research (with greater openness and spillovers) is due to ‘efficiency’ and not to free-riding. Free-riding reduces the growth rate by reducing world knowledge spillovers, since foreign knowledge accumulation is dependent on domestic knowledge accumulation, and vice versa.

¹³ An alternative formulation is where $F_i = F_i(A_p + \theta^* A_p^*, A_g + \theta^* A_g^*)$ and $F_i^* = F_i^*(A_p^* + \theta A_p, A_g^* + \theta A_g)$, $i = p, g$. Here knowledge stocks enter directly in the other country's knowledge production functions. This would be plausible if such stocks were somehow transportable or if research labs were multinational networks. But as long as government research is essential to research (i.e. $\partial^2 F_i / \partial A_p \partial A_g \neq 0$, $\partial^2 F_i^* / \partial A_p^* \partial A_g^* \neq 0$), the results of this section remain intact.

¹⁴ The model could have been made more complicated by allowing for cross-national patenting, for example, allowing domestic research output to be used to create a foreign durable input. Another variation is to allow countries to export and import durable inputs. These would make the model richer but would not change the essential results, particularly since symmetry is imposed everywhere except in the spillover parameters.

¹⁵ See Rivera-Batiz and Romer (1991) for an analysis where redundant research can occur. To the extent that some research in both countries is redundant, the *effective* knowledge base for the home private research sector would be $F_p + \theta^*(F_p^* - I)$ where I is the intersection $F_p \cap F_p^*$, and so forth for the other research sectors.

3.1. Case 1: Symmetric spillovers ($\theta = \theta^*$)

This case is fairly easy to characterize. The results are analogous to those obtained from the closed-economy analysis since the two-country world collapses, under perfect symmetry, into a single world model. The equilibrium balanced growth rate, though, is higher than that of the closed economy. In the case where $\theta = \theta^* = 1$, the common equilibrium balanced growth rate is given by Eq. (9) with H replaced by $2H$. In the case where $\theta = \theta^* < 1$, H is replaced by $H(1 + \theta)$. The magnification of H reflects the fact that human capital gets to work with a larger knowledge base. The growth-maximizing tax rule, or efficiency condition, is still given by Eq. (10).

However, while the nature of the efficiency condition for maximizing growth is the same, the growth-maximizing tax rate is not the same – the rate is lower in the open economy. This can be shown as follows: first, a fully integrated world economy with international knowledge spillovers gives each country a larger *effective* stock of human capital. As the Appendix shows, the increase in H causes an increase in q , the ratio of the stock of private knowledge (A_p) to the stock of public knowledge (A_g), i.e. $dq/dH > 0$. The increase in q in turn causes a decrease in the average productivity of the stock of private knowledge in private research (that is, a decrease in F_p/A_p , where $F_p/A_p = q^{\beta_p-1}$). Consequently, in Eq. (10), the LHS < RHS, so that a decrease in τ is needed to restore the efficiency condition – hence the decrease in *efficient* size of government research (G/Y) in the open economy.

The intuition is that international knowledge spillovers add to what is regarded as ‘publicly’ available knowledge. Both spillover knowledge and government research knowledge are non-excludable, non-rival inputs into the production of new domestic knowledge. While the two are not perfect substitutes, domestic government research is costly (in terms of distortionary taxation). Furthermore, since government research does not directly contribute to new marketable products, but only to the accumulation of further publicly available knowledge, the increased availability of ‘public’ knowledge (from spillovers) results in a shift in the composition of national research in favour of a greater share of private research over public. Hence there is increase in equilibrium q .

The results of Case 1 are summarized as follows:

Proposition 1. *In a symmetric two-country world economy, where the effective stock of human capital in each country is higher, the balanced world equilibrium growth rate is higher and the efficient (growth-maximizing) size of government in each country is smaller.*

It appears that government research size and growth are negatively correlated. However, this is not the result of any direct causation, but rather of a third factor: increased openness and enjoyment of international knowledge spillovers.

3.2. Case 2: Asymmetric spillovers ($\theta \neq \theta^*$)

The two countries are identical except for differences in the magnitude of cross-national spillovers. An example would be where Europe receives more knowledge spillovers from the U.S. than the U.S. receives from Europe, or vice versa. First, some

additional modifications to the model are described. Let $Q_p = (F_p + \theta^* F_p^*)/A_p$, $Q_g = (F_g + \theta^* F_g^*)/A_g$, $Q_p^* = (F_p^* + \theta F_p)/A_p^*$, and $Q_g^* = (F_g^* + \theta F_g)/A_g^*$ be the *effective* average (knowledge output) productivities of the various domestic and foreign stocks of private and public research knowledge.

With international financial capital mobility (and perfect substitutability of financial assets):

$$(1 - \tau)r = (1 - \tau^*)r^* = r_Z \quad (13a)$$

The consumer's intertemporal optimization conditions are:

$$\sigma g + \rho = (1 - \tau)r = (1 - \tau)(1 - \alpha)\delta H_Y Q_p \quad (\text{Home}) \quad (13b)$$

$$\sigma g + \rho = (1 - \tau^*)r^* = (1 - \tau^*)(1 - \alpha)\delta H_Y^* Q_p^* \quad (\text{Foreign}) \quad (13c)$$

where $g = \dot{C}/C = \dot{A}_p/A_p = \dot{A}_g/A_g = \dot{A}_p^*/A_p^* = \dot{A}_g^*/A_g^*$ is the common world balanced growth rate.

This growth rate can be expressed as functions of either the home country's variables or the foreign's. Using Eqs. (12a), (12b), (12c) and (12d), Eqs. (13a), (13b) and (13c), and the human capital resource constraints in each country, gives:

$$g = \frac{(1 - \tau)(1 - \alpha)\delta H - \rho(1/Q_p)}{(1 - \tau)(1 - \alpha)(1/Q_p + 1/Q_g) + \sigma(1/Q_p)} \quad (14a)$$

Both equations are similar in form to Eq. (9) derived for

$$g = \frac{(1 - \tau^*)(1 - \alpha)\delta H - \rho(1/Q_p^*)}{(1 - \tau^*)(1 - \alpha)(1/Q_p^* + 1/Q_g^*) + \sigma(1/Q_p^*)} \quad (14b)$$

the closed economy. The spillover parameters are embedded in the Q 's (the effective average productivities of knowledge capital). The larger the cross-national spillovers (that is, the higher θ and θ^*), the greater the world balanced growth rate, as the effective stock of human capital in each country becomes larger. In Eq. (14a), holding τ^* constant, the growth rate g is concave downward in τ as long as $q = (A_p/A_g)$ is decreasing in τ and τ^* at a diminishing rate (a condition analogous to the closed-economy case).¹⁶ Likewise, g is concave downward in τ^* (holding τ constant), under similar assumptions.

The pair of domestic and foreign tax rates which maximizes the world balanced growth rate is denoted by (τ_c, τ_c^*) . This pair satisfies the following efficiency conditions:

$$\frac{\Delta Q_p}{Q_p} = \frac{\Delta \tau}{\tau} \frac{\tau}{1 - \tau} (1 + \gamma_1 \gamma_2) \quad (15a)$$

$$\frac{\Delta Q_p^*}{Q_p^*} = \frac{\Delta \tau^*}{\tau^*} \frac{\tau^*}{1 - \tau^*} (1 + \gamma_1^* \gamma_2^*) \quad (15b)$$

¹⁶ That is, increase in τ^* increase $(1/q)$ at a diminishing rate because eventually they slow the growth of foreign innovations, which in turn slow the growth of domestic.

where

$$\gamma_1 = \frac{(1-\tau)\tau}{\alpha} Q_g > 0, \quad \gamma_2 = \frac{\partial}{\partial \tau} \left[\frac{1}{Q_p} + \frac{1}{Q_g} \right] > 0$$

$$\gamma_1^* = \frac{(1-\tau^*)\tau^*}{\alpha} Q_g^* > 0, \quad \gamma_2^* = \frac{\partial}{\partial \tau^*} \left[\frac{1}{Q_p^*} + \frac{1}{Q_g^*} \right] > 0$$

Condition (15a) is obtained by setting $\partial g / \partial \tau = 0$ (using Eq. (14a)) and condition (15b) by setting $\partial g / \partial \tau^* = 0$ (using Eq. (14b)). The economic interpretation of these conditions is similar to that of Eq. (10). The LHS represents the marginal benefits of government research expansion in terms of the growth in the *effective* average productivity of the stock of private knowledge in private research. The RHS represents the marginal cost of government research expansion in terms of the growth in distortions.

It is also the case that $\tau_c \leq (\geq) \tau_c^*$ according to $\theta^* \geq (\leq) \theta$. As discussed in the symmetric two-country case, the greater the stock of spillover knowledge, the greater the effective stock of human capital, and the lower the optimal τ . Accordingly, the country which receives more international knowledge spillovers has the larger effective stock of human capital, the higher q ratio, and thus the lower growth-maximizing tax rate or size of government research.

Now holding foreign τ^* constant, a domestic τ exists which maximizes the common world balanced growth rate g . The Appendix shows that $d\tau/d\tau^* < 0$, that is, as foreign τ^* increases, the critical value τ (at which $\partial g / \partial \tau = 0$) decreases. Fig. 2 plots the common world balanced growth rate against the domestic tax rate, holding different values of τ^* constant. So long as the foreign tax rate is between zero and τ_c^* , unilaterally raising τ^* towards τ_c^* raises the ‘growth’ curve so that every domestic tax rate is associated with a higher common world balanced growth rate. However, raising foreign τ^* when $\tau^* > \tau_c^*$, causes the growth curve to shift down. Also, as stated, the critical or growth-maximizing domestic tax rate decreases as τ^* increases. For example, if $0 < \tau^* < \tau_c^*$, the critical domestic tax rate will exceed τ_c . The world balanced growth rate will not be maximized if the domestic tax rate equals τ_c so long as $\tau^* \neq \tau_c^*$. But as the foreign tax rate is raised towards τ_c^* , lowering the domestic tax rate towards τ_c will increase the world balanced growth rate. The maximum rate is achieved when $\tau = \tau_c$ and $\tau^* = \tau_c^*$. The negative correlation between the foreign tax rate and domestic critical tax rate is attributable to there being an optimal amount of government research for the world economy, just as there was one for the closed economy. If the foreign government’s research size is inefficiently large, the constrained optimal solution for the domestic government is to reduce its research size below that which is associated with τ_c .

The fact that one nation’s size of government research falls as the other’s increase appears as if the former is free-riding. Rather, the motive is one of efficiency. Efficiency-minded governments respond inversely to changes in public research size abroad so as to maintain an overall world public research size that maximizes growth.

There is another sense in which efficient government behavior precludes free-riding. To illustrate, suppose the domestic economy is at point B in Fig. 2. From this point there is no way to achieve a higher growth rate unless the foreign government expands its

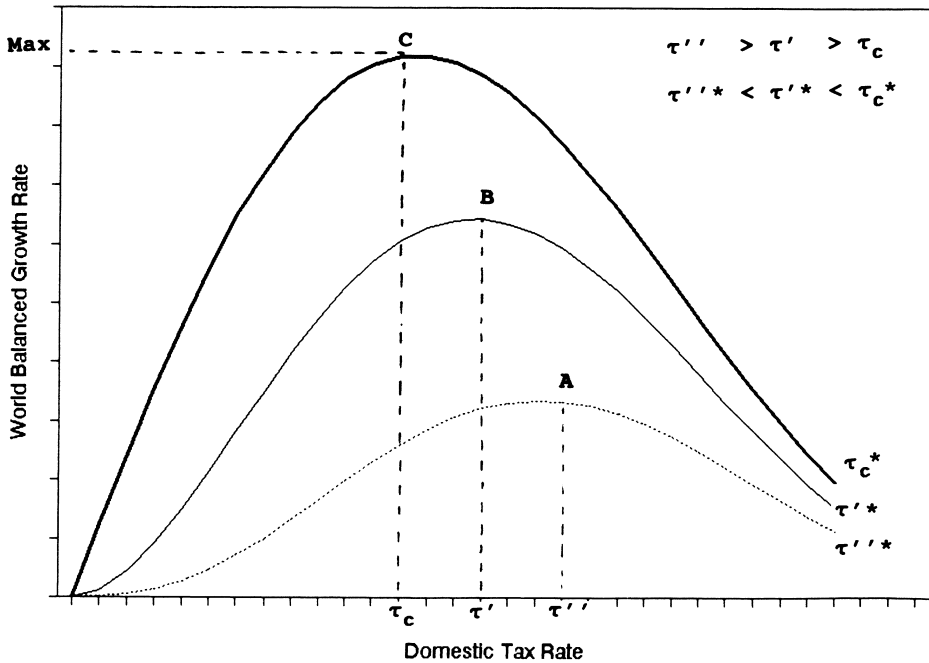


Fig. 2. Balanced growth rate open economy, asymmetric spillovers.

research. A unilateral lowering or raising of the domestic tax rate would lead to a slower long run growth rate. While the domestic economy may continue to enjoy foreign research spillovers, doing less than the efficient amount of research at home has adverse repercussions. The reason can be seen from Eqs. (12a), (12b), (12c) and (12d): domestic and foreign research influences each other. Inefficiently reducing domestic research reduces spillovers from domestic into foreign research. The resulting decline in foreign research then reduces spillovers from foreign into domestic research. Likewise, inefficiently expanding domestic research feeds back to the domestic economy via its impact on foreign research.

This simple illustration reflects a stylized fact that, despite the great potential for free-riding on worldwide research activities, R&D nations continue to engage in R&D. Perhaps, non-R&D nations conduct no research not because they are free-riding but because they are unable to capture the kinds of international research spillovers which make domestic research profitable.

The results of Case 2 are summarized as follows:

Proposition 2. *In a two-country world with asymmetric spillovers, a critical pair of domestic and foreign tax rates maximizes the world balanced growth rate. The domestic tax rate is lower (higher) than the foreign tax rate if the domestic economy receives a higher (lower) rate of spillovers than the foreign. Free-riding on international research spillovers is inefficient in a world of interdependent knowledge accumulation.*

4. Conclusions

This paper extends recent work on knowledge capital accumulation and growth to incorporate public sector research. The long run growth rate is shown to depend on both excludable and non-excludable stocks of knowledge, including international spillover knowledge. The results suggest that in an integrated world economy, certain misleading inferences might be made about government efficiency and behavior. For example, under increased openness, a country's growth rate rises and government research size decreases. This gives the impression that government research has a negative effect on growth. Secondly, an increase in the size of foreign government research lowers the size of domestic government research. This gives the impression that governments free-ride on foreign research. In both cases, however, the reduction in government research size is the appropriate, optimizing growth response to an increase in foreign knowledge spillovers. With more 'publicly' available knowledge, a smaller *public* research size is efficient (for maximizing growth). This is conditional on the assumption that government research does not directly contribute to new marketable products, but only to further knowledge accumulation.

To conclude, it is worth examining how the model fits with the trends shown in Fig. 1. While a more controlled investigation is required to explain them,¹⁷ the theoretical model nonetheless has some relevant time-series and cross-sectional implications. On the time-series side, Fig. 1 shows a declining share of total R&D performed in the government sector. In terms of the model, this change in the composition of national research increases q (the ratio of private to public knowledge capital). Of course, governments might just be responding to pressures to reduce their budgets, regardless of the implications for productivity. But if governments were optimizing (say, maximizing the growth rate), the increase in q would, according to the model, be an 'optimizing' response to: (i) an increase in Λ_2 (or γ_2), which occurs if private R&D is more productive in generating new innovations (as this raises the opportunity cost of human capital resources in the public sector) or (ii) more 'openness' (in the sense of enjoying more international knowledge spillovers), since the model predicts that more open economies have smaller government research sectors.

Cross-sectionally, Fig. 1 indicates that small to medium R&D countries have a larger share of government research than large R&D countries. Now, the analysis so far has emphasized the non-excludable character of government research. However, as Romer (1990) stresses, excludability is a function of both the legal system and the nature of the underlying commodity. This suggests another interpretation of the model: countries which have less strongly enforced intellectual property rights have a broader continuum of 'designs' that are not excludable and thus rely more on public R&D. Thus, public R&D can (at least partly) be a substitute for a weak patent system. Indeed in studies of patent regimes, smaller R&D nations are found to provide weaker patent protection than the larger R&D nations.¹⁸ In terms of the model, countries whose private research sector

¹⁷ For one thing, the figure refers to government research (G) as a share of total research, whereas the model refers to G as a share of the entire economy (Y). The two measures correlate positively as long as private research is held constant.

¹⁸ See Ginarte and Park (1997) and Rapp and Rozek (1990).

is less able to commercialize new innovations (due to weaker patent protection) are likely to have a lower Λ_2 (or γ_2) and thus a relatively larger share of public R&D.

There is also another reason why government research shares should differ across countries. As the model suggests, nations that enjoy more foreign knowledge spillovers should *ceteris paribus* have smaller government research sectors. Between large and small R&D nations, it might appear that smaller nations are likely to receive more spillovers from the larger R&D nations than vice versa because the knowledge bases of the larger R&D nations are much larger. However, this is offset by the fact that the relevant parameter (namely θ or θ^*) is likely to be much smaller for the smaller R&D nations. The spillover parameters tend to be functions of the capacity to absorb foreign knowledge (which in turn depends on how much R&D the country itself does and on its technological similarity with other R&D nations).¹⁹ Hence, larger R&D nations are likely to enjoy more international knowledge spillovers. This should contribute to their having smaller government research sectors.

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Appendix

Derivation of (dq/dH)

Substitute $g = \delta H_p F_p / A_p$ into Eq. (9). From Eqs. (8a) and (8b), $H_g / H_p = (F_p / A_p) / (F_g / A_g)$ under balanced growth. Using this and Eqs. (5) and (6), the equilibrium allocation of human capital can be determined. (Y is eliminated using the expression for p_A – see Footnote 9). Substituting the allocations of human capital into Eq. (9) yields an equation in which to relate q to the tax rate, τ :

$$\left[\frac{A_p}{F_p} + \frac{A_g}{F_g} \left(1 + \frac{\alpha}{\tau} \right) \right] \rho = \left[\left(\frac{1}{\tau} - 1 \right) \alpha (1 - \alpha) \frac{F_p A_g}{A_p F_g} - \sigma \right] \delta H$$

where $(A_p / F_p) = q^{1-\beta_p}$ and $(A_g / F_g) = q^{\beta_g-1}$.

From this latter equation:

$$\text{sgn} \left(\frac{dq}{dH} \right) = \text{sgn} \left[\left(\frac{1}{\tau} - 1 \right) \alpha (1 - \alpha) \frac{F_p A_g}{A_p F_g} - \sigma \right] > 0$$

(ii) Derivation of $(d\tau/d\tau^*)$

¹⁹ See Park (1995).

Using (26a),

$$\frac{d\tau}{d\tau^*} = \left[\frac{\partial^2 Q_p}{\partial \tau \partial \tau^*} \frac{1}{Q_p} - \frac{1}{Q_p^2} \frac{\partial Q_p}{\partial \tau} \frac{\partial Q_p}{\partial \tau^*} + \frac{\tau Q_g}{\alpha} \left[\frac{\partial^2 Q_p}{\partial \tau \partial \tau^*} \frac{1}{Q_p^2} - \frac{2}{Q_p^3} \frac{\partial Q_p}{\partial \tau} \frac{\partial Q_p}{\partial \tau^*} + \frac{\partial^2 Q_g}{\partial \tau \partial \tau^*} \frac{1}{Q_g} \right] \right. \\ \left. + \frac{\tau}{\alpha} \frac{\partial Q_p}{\partial \tau} \frac{\partial Q_g}{\partial \tau^*} - \frac{\tau}{\alpha} \frac{1}{Q_g^2} \frac{\partial Q_g}{\partial \tau} \frac{\partial Q_g}{\partial \tau^*} \right] \left[\frac{1}{(1-\tau)^2} + \frac{Q_g}{\alpha} \gamma_2 \right]^{-1} < 0$$

as the numerator is negative and denominator positive. Note that $\partial^2 Q_p / \partial \tau \partial \tau^*$ and $\partial^2 Q_g / \partial \tau \partial \tau^*$ are negative if $\partial^2 q / \partial \tau \partial \tau^* > 0$ (that is, under diminishing returns to expanding domestic government research, an increase (decrease) in *foreign* government research decreases (increases) the marginal benefits of expanding *domestic* government research).

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