# Real Options in Patenting: Role of Secondary Patent Markets

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# Introduction

Economic Upheavals

- Coronavirus Pandemic 2020 -
- Great Recession 2008-9
- Internet Bubble 2000-1

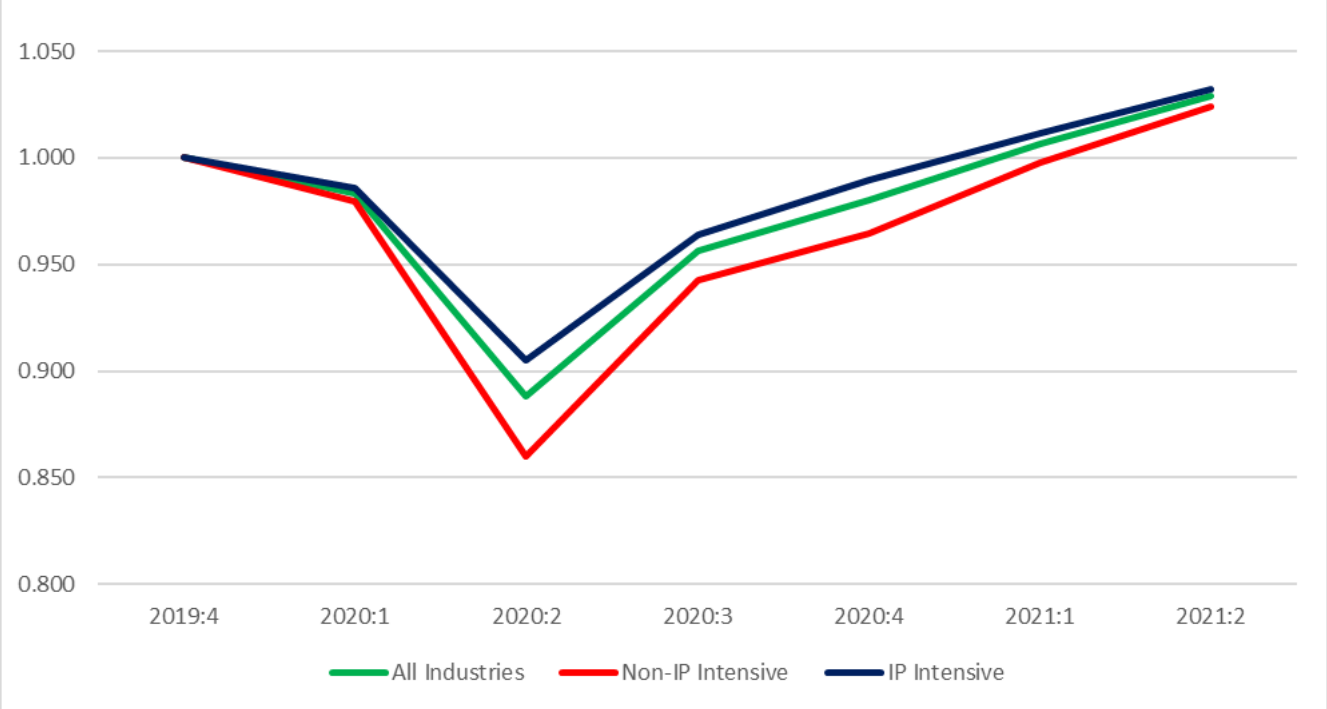
Innovation resilience

Takeaways during uncertain times:

- Declines in output in IP-industries *fared better* than in non-IP-industries
- Investments in knowledge can be repurposed. They're **not** necessarily **sunk** costs.
- Patent assets are *tradable* (particularly in the digital technology fields).
- Patent protection allows for *reversible* investment – that is, companies can use it to store value and recoup investments.

# Covid-19 Shock: U.S.

Figure 1: U.S. Real Gross Output

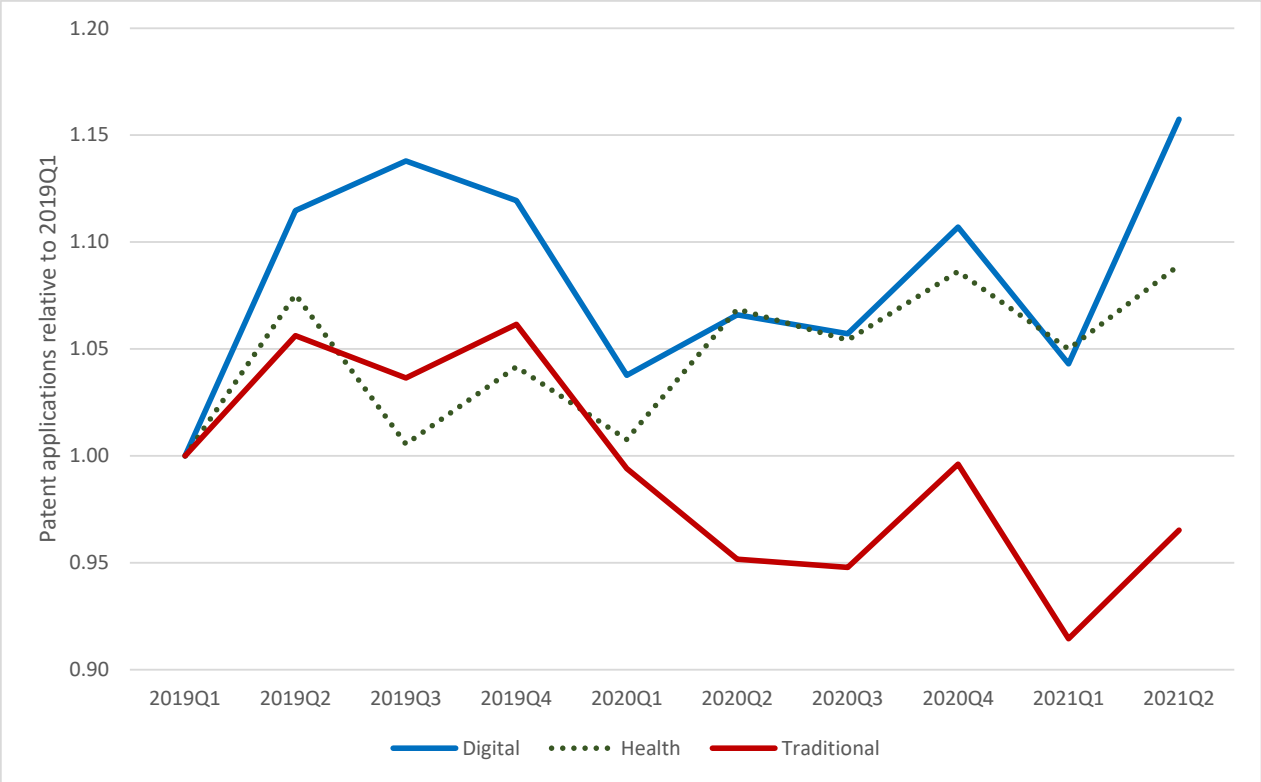



Gross real output by industry. The gross output measure, seasonally adjusted at annual rates, is used, and converted into real 2012 dollars using its chain-type price index.

Source: Authors' calculations based on *U.S. Bureau of Economic Analysis* Interactive Tables (www.bea.gov).

# Patenting by Sector

## Figure 2: Patent Filings from US-based Inventors by Industry Group




Notes: Utility (invention) patent applications filed by receipt date, the date which the application is initially submitted to USPTO. The first-listed CPC symbol for each application is used to classify each application. Origin counts are fractionalized based on each inventor's country code in their mailing address.

Source: Author's calculations based on data from USPTO's Patent Location and Monitoring system (PALM).

# Secondary Market Deals

## Table 1: Recent deals on IAM

**SECONDARY MARKET DEALS IN PATENT RIGHTS: INDUSTRY REPRESENTATION**

| Industry group | Counts | Share (%) |
|---|---|---|
| Digital technologies | 393 | 44.0 |
| Health technologies | 116 | 13.0 |
| Automotive and parts | 65 | 7.3 |
| Telecommunications | 46 | 5.2 |
| Manufacturing | 46 | 5.2 |
| Media & entertainment | 31 | 3.5 |
| Electricity | 27 | 3.0 |
| Transport | 27 | 3.0 |
| Security & defence | 26 | 2.9 |
| Household goods | 22 | 2.5 |
| Chemicals | 13 | 1.5 |
| Food, tobacco, & beverages | 13 | 1.5 |
| Oil & gas | 10 | 1.1 |
| Finance | 10 | 1.1 |
| Retail & distribution | 10 | 1.1 |
| Construction & building materials | 7 | 0.8 |
| Test and measurement | 7 | 0.8 |
| Support services | 6 | 0.7 |
| Steel & other metals | 5 | 0.6 |
| Utilities & mining | 5 | 0.6 |
| Leisure & hospitality | 4 | 0.4 |
| Printing | 3 | 0.3 |
| Education | 1 | 0.1 |
| **TOTAL** | **893** | **100.0** |

Note: Counts of transactions for all technology categories. Each transaction, or deal, consists of one or more patents.
Source: IAM Market (https://portal.iam-market.com/browse-iptech, accessed 21 March 2022).

# Outline

Goals:

- Test and apply *real options theory* to patenting under uncertainty
- Measure (i) economic uncertainty in different ways and (ii) exposure to secondary patent markets

Plan of Talk:

- Review real options concepts
- Present evidence
  - Empirical framework
  - Measures of uncertainty and Index of Secondary Patent Market exposure
  - Data sources
  - Findings
- Discuss implications and way forward

Key findings:

- Patenting is normally *negatively* associated with uncertainty.
- But for firms with sufficient access to secondary patent markets, increased uncertainty could spur greater patenting

# Related Literature

Uncertainty and investment

- Abel et al. (1996) "Options, the Value of Capital, and Investment." QJE
- Bloom et al. (2022) "Investment and Subjective Uncertainty." NBER
- Bulan (2005) "Real Options, Irreversible Investment and Firm Uncertainty." RFE
- Dixit and Pindyck (1994) *Investment Under Uncertainty*. PUP

Business shocks, uncertainty, and innovation

- Bloom and Van Reenan (2000) "Patents, Real Options, and Firm Performance." EJ
- Czarnitzki and Toole (2011) "Patent Protection, Market Uncertainty, and R&D ..." ReSTAT
- Ghosal and Loungani (2000) "Differential Impact of Uncertainty ..." ReSTAT
- Hingley and Park (2017) "Do Business Cycles Affect Patenting? ..." TFSC

Covid effects on business, employment, and innovation

- Buffington et al. (2021) "High Frequency Business Dynamics ..." Census Bureau
- Davis et al. (2021) "STEM Employment Resiliency ..." NBER
- Dinlersoz et al. (2021), "Business Formation ..." Census Bureau

Secondary markets for patents

- Akcigit et al. (2016) "Buy, keep, or sell ... market for ideas." ECTA
- Arora et al. (2022) "Science and market for tech" MS
- Lamoreaux and Sokoloff (2003) "Intermediates ..." in Engerman et al. (eds.)
- Love et al. (2018) "An Empirical Look at the 'Brokered' Market ..." MLR
- Serrano (2018) "Estimating the Gains from Trade in the Market for Patent Rights." IER

Patent collateral

- Barontini and Taglialatela (2022) "Patents and Small Business Risk ..." JSBED
- Bracht and Czarnitski (2022) "Patent Collateral ..." ZEW DP
- Mann (2018) "Creditor Rights and Innovation ..." JFE

# Real Options Theory: Overview

Developed for Real Investments ... but is applicable to Intangible Investments

- Future patent holdings can be expanded or contracted
- Future returns may be uncertain
  - When firms can resell patents at a later date, it possesses a *put option*
  - When firms can acquire patents at a later date, it possesses a *call option*
  - Both options affect current incentives to invest
  - Higher values of the call option create incentives to postpone patenting, while higher values of the put option create incentives to patent currently
- Greater future uncertainty has an ambiguous effect on investments in patent assets because it increases the value of both the put and call options.
- Patent resale markets allow innovation investments to be reversible. The more reversible, the greater the incentive to innovate and patent.

# Conceptual Framework

## Adaptation of Abel et al. (1996)

Our purpose:

1. show why uncertainty has an ambiguous effect on patenting
2. explain role of a secondary patent market

Two-period model: firm chooses patent portfolio, $x$, to max firm value $V(x) = \pi_1(x) + \phi\pi_2^e(x, \boldsymbol{\sigma})$ s.t. rx.

Two critical values of the shock: $\frac{\partial \pi_2}{\partial x}(x, \boldsymbol{\sigma_L}) = r_L$ and $\frac{\partial \pi_2}{\partial x}(x, \boldsymbol{\sigma_H}) = r_H$, where $r_L \leq r_H$.

Let $F(\sigma)$ be the CDF of $\sigma$ with support $\mathbb{R}$. $r_L$ = resale price of a patent and $r_H$ = purchase price of a patent in period 2. Let $x'$ = patent portfolio in period 2, and $\Delta x = (x' - x) \gtreqless 0$ if sale/purchase, and zero otherwise.

Period 2 returns:

$$(1) \qquad \pi_2^e = \int_{\sigma_L}^{\sigma_H} \pi_2(x, \sigma) dF(\sigma) + \int_{\sigma_H}^{\infty} (\pi_2(x', \sigma) - r_H \Delta x) dF(\sigma) + \int_{-\infty}^{\sigma_L} (\pi_2(x', \sigma) + r_L \Delta x) dF(\sigma)$$

Substitute (1) into $V(x)$, and rearrange. $\Longrightarrow$

# Conceptual Framework

Firm value is the sum of three parts:

$$V(x) = R(x) + P(x) - C(x) \tag{2}$$

where

$$R(x) = \pi_1 + \phi \int_{-\infty}^{\infty} \pi_2(x, \sigma) dF(\sigma) \tag{3}$$

$$P(x) = \phi \int_{-\infty}^{\sigma_L} (\pi_2(x', \sigma) - r_L x') - (\pi_2(x, \sigma) - r_L x) dF(\sigma) \tag{4}$$

Put option

$$C(x) = \phi \int_{\sigma_H}^{\infty} -(\pi_2(x', \sigma) - r_H x') + (\pi_2(x, \sigma) - r_H x) dF(\sigma) \tag{5}$$

Call option

**F.O.C.**

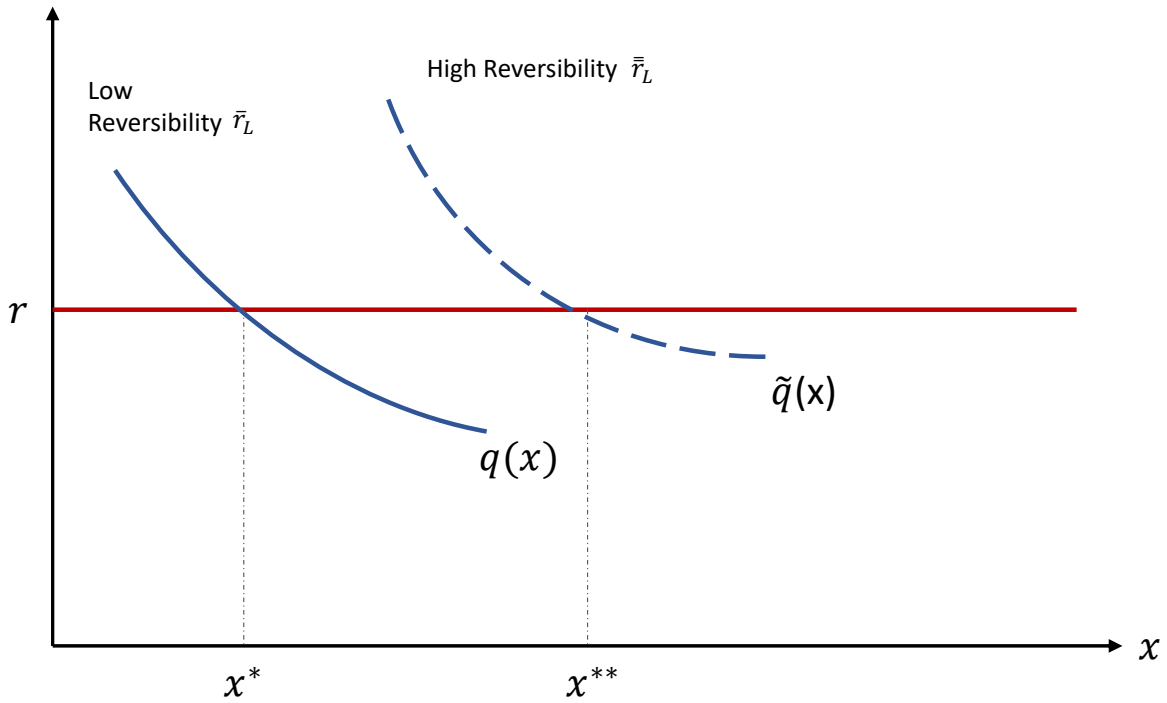
$$\underbrace{R'(x) + P'(x) - C'(x)}_{=q(x)} = r$$

where $P'(x) = \phi \int_{-\infty}^{\sigma_L} (r_L - \frac{\partial \pi_2}{\partial x}(x, \sigma)) dF(\sigma)$ and $C'(x) = \phi \int_{\sigma_H}^{\infty} (\frac{\partial \pi_2}{\partial x}(x, \sigma) - r_H) dF(\sigma)$

And importantly, $\frac{\partial q(x)}{\partial r_L} = \phi F(\sigma_L) \geq 0$

# Marginal Value of Patents in Period 1, q(x)

Figure 3: Different Resale Prices, $\bar{\bar{r}}_L > \bar{r}_L$




$q(x)$ is decreasing in x, as $R'$, $P'$, and $C'$ are decreasing in x.

# Conceptual Framework

Effect of Uncertainty, $\sigma$

Mean-preserving spread in distribution of $\sigma$ increases both $P'(x)$ and $C'(x)$
(i.e., weights of both upper and lower tails of the CDF of the marginal return to x are increased)

$\therefore$ Effect on $x$ is ambiguous.
(More valuable put options motivate patenting while more valuable call options create incentives for postponement.)

However, higher values of $r_L$ (greater reversibility), holding other factors constant, raise weight of put options.

Leading us to test:

**Hypothesis 1 (H1)** *Patenting decreases with uncertainty about future returns.*

**Hypothesis 2 (H2)** *For firms with sufficient access to secondary markets, patenting increases with uncertainty about future returns.*

# Estimating Equation

Specification of Model:

$$P_{it} = exp(\alpha_0 + \alpha_i + \alpha_t + \beta_1 UNC_{it} + \beta_2 (UNC_{it} \times ISE_i) + \mathbf{X}'_{it}\delta)\varepsilon_{it},$$

- Unit of analysis is company $i$ at quarter $t$
- $P_{it}$ denotes patent grants by date of application
- $UNC_{it}$ is the level of uncertainty faced by firm $i$ at time $t$
- $ISE_i$ is the long term exposure of firm to the secondary patent market.
- $\mathbf{X}_{it}$ is the vector of time-varying firm controls: R&D, sales, ratio of cash flow to capital stock, assets, firm age, industrial concentration (HHI of sales share in industry), and technology specialization (HHI of CPC code share in firm's patent portfolio).
  Not all reported in tables below to conserve space
- $\alpha_i$ and $\alpha_t$ are firm and quarter fixed effects.
- $\varepsilon_{it}$ is the stochastic error term.

Exponential Model ($exp$) is estimated by PPML (w/ high-density fixed effects). Method of estimation is robust to any heteroskedasticity in $\varepsilon_{it}$ and to zero counts of $P_{it}$.

Hypothesis: $\beta_1 < 0$ and $\beta_2 > 0$

# Sample: Companies filing at USPTO

Firm Level Data: 1990 - 2021, quarterly

1. Major Patenting Firms in the United States:
   - 800 companies *(accounting for 33% of all U.S. grants)*
   - 15 countries represented, including U.S.
   - 94 industries, NAICS 4-digit level
2. Data Sources:
   - American Inst. for Research and USPTO *PatentsView*
   - Standard & Poor *Capital IQ*
   - Walls & Assoc. *National Establishment Times-Series (NET)* database
   - IAM market, Richardson-Oliver Insights (ROI)
   - U.S. Bureau of Economics Analysis *Data by Economic Accounts*
3. Data Preparation:
   - Firm name disambiguation
   - Concordance: match assignees/subsidiaries to parent companies
   - Match patenting data to firm level data in *Capital IQ*
   - Assignment records not available in pre-grant data. Hence, we use data on patent grants as of the date of filing.
   - All relevant data converted to real 2012 dollars

# Alternative Approaches to Measuring Uncertainty and Shocks

(1) Coefficient of variation:

$$UNC_{it} = \frac{\sqrt{var(z_{it})}}{mean(z_{it})}$$

where $z_{it} = \dfrac{\frac{sales_{it}}{employees_{it}}}{\frac{1}{N^{\iota}} \sum_{i=1}^{N^{\iota}} \frac{sales_{it}}{employees_{it}}}$

and $N^{\iota}$ = number of firms in industry $\iota$.

(2) Auto-regressive model:

$$z_{it} = \gamma_0 + \gamma_1 z_{it-1} + \gamma_2 z_{it-2} + \gamma_3 z_{it-3} + \gamma_4 z_{it-4} + \gamma_t t + \epsilon_{it}$$

From the standard deviation of the residuals: $UNC_{it}^{AR} = \sigma(\epsilon_{it})$.

# Alternative Approaches to Measuring Uncertainty and Shocks

(3) Stock Market Volatility:

Let $S$ denote stock price and $\rho = \frac{\Delta S + \text{DIV}}{S}$ the rate of return. Regress firm rate of return on those of the market and industry (where $\rho_{I\tau}$ is made orthogonal to $\rho_{M\tau}$):

$$\rho_{i\tau} = \psi_{it} + \psi_{it}^{M}\rho_{M\tau} + \psi_{it}^{I}\rho_{I\tau} + \epsilon_{i\tau}$$

where $\tau$ indexes trading day.

Construct the quarterly standard deviation of the residuals, $\hat{\epsilon_{i\tau}}$: $UNC_{it}^{S} = \sqrt{n}\ \sigma(\rho_{i\tau})$, where $n =$ number of trading days per quarter.

(4) Cyclical-Trend Filtering (Baxter-King method):

Let $z_{it} = z_{it}^{T} + z_{it}^{C}$, where $z_{it}^{T}$ denotes the trend level of real sales per worker and the $z_{it}^{C}$ the cyclical component.

Business shock variable, $BK_{it} = \frac{z_{it}^{C}}{z_{it}^{T}}$

# Brokered Markets

Using data on patent 'packages', we obtain **prices** at which packages are sold and determine **technological fields (CPC codes)** covered by the patents.

We exploit idea that (a) some technological fields are more involved than others in trading and (b) firms vary in the technological fields specified in their patent applications.

The *Index of Secondary Patent Market Sales Exposure, ISE* for firm $i$ is:

$ISE_i = \sum_{j=1}^{N} Share_{ij} \times Proportion_j$

- j indexes technological field and N the total number of technological fields the firm innovates in during a given sample period.
- $Share_{ij}$ is the share of the $j^{th}$ technological field in the firm's overall patent grants.
- $Proportion_j$ is the share of the $j^{th}$ technological field in all the patent packages.

Two versions:

1. $Proportion_j$ is *unweighted*, based on raw counts: $\frac{CPC_j}{\sum_j CPC_j}$
2. $Proportion_j$ is *weighted*, by value of an imputed price for field $j$: $\frac{p_j CPC_j}{\sum_j p_j CPC_j}$

Table 2: Top 12 CPC subclasses in the share of secondary market sales of granted patents

| CPC Sub Class | % share total value | CPC_SubClass_Title |
|---|---|---|
| H04 | 43.29 | Electric Communication Technique |
| G06 | 27.20 | Computing; Calculating; Counting |
| H01 | 6.62 | Basic Electric Elements |
| G01 | 2.98 | Measuring; Testing |
| H03 | 2.67 | Basic Electronic Circuitry |
| G11 | 2.07 | Information Storage |
| G02 | 1.69 | Optics |
| G08 | 1.67 | Signaling |
| A61 | 1.52 | Medical or Veterinary Science; Hygiene |
| G10 | 1.50 | Musical Instruments; Acoustics |
| H05 | 1.01 | Electric Techniques not Otherwise Provided for |
| H02 | 0.90 | Generation; Conversion, Distribution of Electric Power |

Table 3: Sample Indexes of Secondary Market Sales, by Industry

| NAICS code | Industry | Index of Sales Exposure |
|---|---|---|
| 5182 | Telecomm Data Process. Services | 0.3007 |
| 5112 | Software publishers | 0.2602 |
| 3342 | Communications equipment manuf. | 0.2062 |
| 3343 | Audio and video equipment manuf. | 0.1962 |
| 3341 | Computer and peripheral equipment manuf. | 0.1959 |
| 3344 | Semiconductors and other elect. comp. manuf. | 0.1209 |
| 5324 | Commercial and Industrial Machinery... | 0.0720 |
| 2211 | Electric Power Gen., Trans, & Distr. | 0.0481 |
| 3364 | Aerospace products and parts manuf. | 0.0347 |
| 3332 | Industrial machinery | 0.0340 |
| 3272 | Glass and glass products | 0.0338 |
| 3391 | Medical equipment and supplies manuf. | 0.0264 |
| 3361 | Motor vehicles manufacturing | 0.0261 |
| 3336 | Engines, turbines, and power trans. equip. manuf. | 0.0199 |
| 3254 | Pharmaceuticals and Medicines | 0.0153 |
| 3262 | Rubber Products | 0.0108 |
| 3261 | Plastic Products | 0.0070 |
| 3311 | Iron and Steel Mills and Ferro alloys | 0.0067 |
| 3251 | Basic Chemicals | 0.0059 |
| 6216 | Home Health Care Services | 0.0010 |

# Index of Secondary Market Exposure (ISE)

Figure 3: Histogram

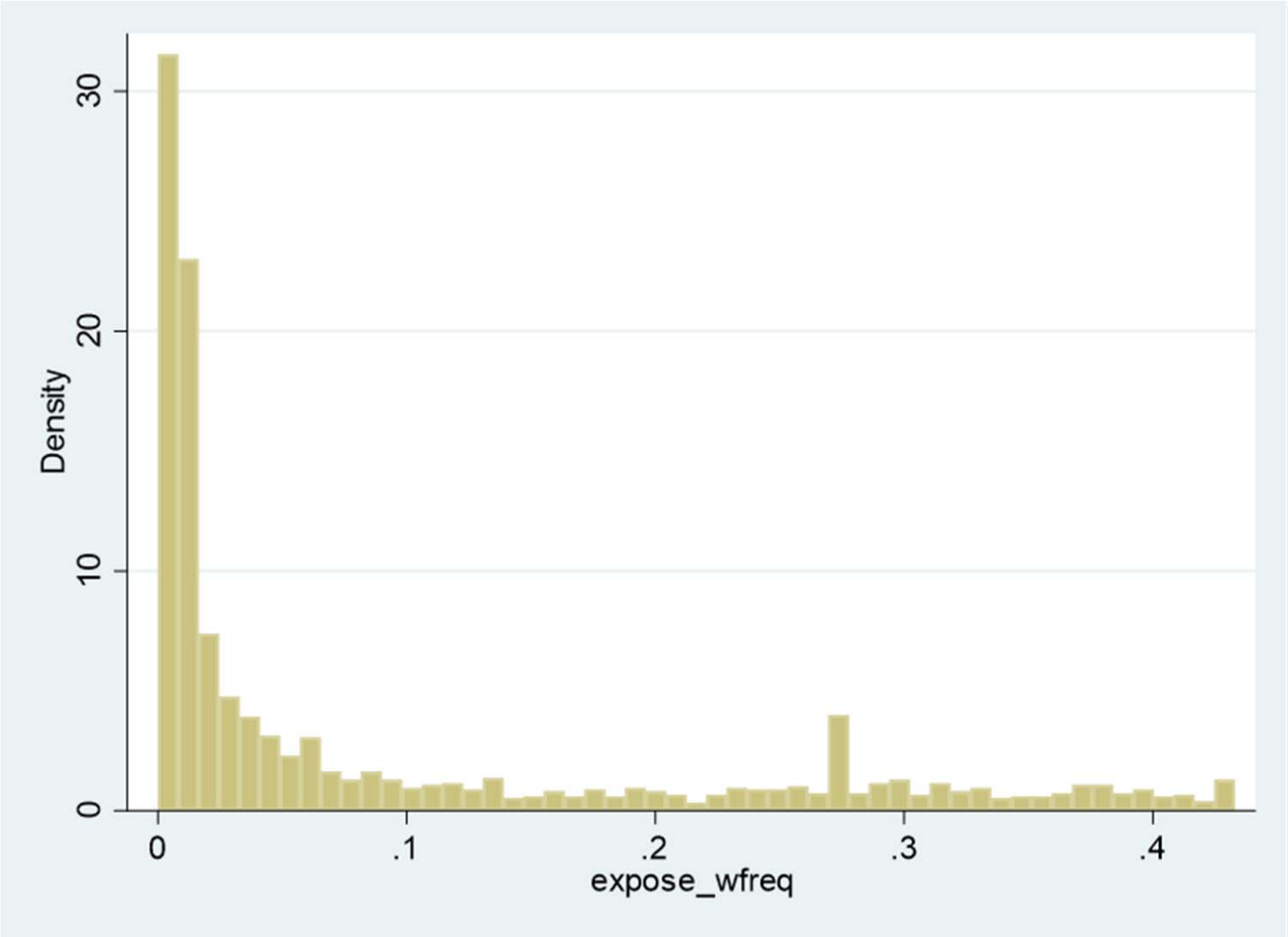


Table 4: Types of Firms by Secondary Market Exposure

| Quintile | Index Sec. Mkt Expo. | Patent Grants | Sales | R&D | FT Employ. | Debt | Age (years) | HHI | Tech Spec. |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.004 | 6.5 | 1007.5 | 53.8 | 10679 | 1579.4 | 52 | 0.283 | 0.604 |
| 2 | 0.014 | 10.5 | 1564.4 | 122.0 | 14719 | 2857.7 | 44 | 0.198 | 0.644 |
| 3 | 0.043 | 21.9 | 1342.5 | 73.0 | 15245 | 2362.4 | 24 | 0.209 | 0.605 |
| 4 | 0.151 | 64.1 | 2283.5 | 187.6 | 23574 | 1895.3 | 36 | 0.191 | 0.519 |
| 5 | 0.333 | 24.7 | 1188.7 | 132.9 | 9021 | 1099.2 | 24 | 0.259 | 0.748 |

Figures represent quarterly averages, unless indicated otherwise.

## Table 5: **Main Results**

| | Grants by Filing Date | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Ln R&D Flow | 0.213*** | 0.214*** | 0.215*** | 0.214*** | 0.219*** | 0.218*** |
| | (0.027) | (0.027) | (0.027) | (0.027) | (0.027) | (0.027) |
| Ln Sales | 0.107** | 0.118*** | 0.114*** | 0.105** | 0.111*** | 0.112*** |
| | (0.042) | (0.042) | (0.042) | (0.042) | (0.042) | (0.042) |
| Indus Concentration | -0.197 | -0.282** | -0.245* | -0.187 | -0.288** | -0.270* |
| | (0.141) | (0.140) | (0.141) | (0.140) | (0.140) | (0.140) |
| Tech Specialization | -0.765*** | -0.784*** | -0.774*** | -0.763*** | -0.788*** | -0.781*** |
| | (0.060) | (0.060) | (0.060) | (0.060) | (0.060) | (0.060) |
| *UNC* (Sales) | -0.157*** | -0.501*** | -0.428*** | | | |
| | (0.054) | (0.110) | (0.122) | | | |
| *UNC* (Sales) × *ISE* | | 2.478*** | | | | |
| | | (0.501) | | | | |
| *UNC* (Sales) × Unweighted *ISE* | | | 1.950*** | | | |
| | | | (0.591) | | | |
| *UNC* (Profit) | | | | -0.147*** | -0.405*** | -0.411*** |
| | | | | (0.049) | (0.095) | (0.101) |
| *UNC* (Profit) × *ISE* | | | | | 1.739*** | |
| | | | | | (0.376) | |
| *UNC* (Profit) × Unweighted *ISE* | | | | | | 1.784*** |
| | | | | | | (0.422) |
| Other CONTROLS | Y | Y | Y | Y | Y | Y |
| Constant | 2.130*** | 2.170*** | 2.149*** | 2.118*** | 2.118*** | 2.112*** |
| | (0.317) | (0.305) | (0.310) | (0.317) | (0.307) | (0.309) |
| Threshold Index | | 0.20 | 0.22 | | 0.23 | 0.23 |
| Pseudo $R^2$ | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 | 0.90 |
| Observations | 23,161 | 23,161 | 23,161 | 23,156 | 23,156 | 23,156 |

## Table 6: **Alternative Measures of Uncertainty**

| | Grants by Filing Date | | | | | |
|---|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) | (6) |
| Ln R&D Flow | 0.285*** | 0.279*** | 0.201*** | 0.200*** | 0.281*** | 0.283*** |
| | (0.027) | (0.027) | (0.025) | (0.025) | (0.026) | (0.026) |
| Ln Sales | 0.101** | 0.094** | 0.138*** | 0.133*** | 0.119*** | 0.115*** |
| | (0.043) | (0.043) | (0.040) | (0.040) | (0.042) | (0.042) |
| $UNC^{AR}$ | -0.111*** | -0.345*** | | | | |
| | (0.029) | (0.077) | | | | |
| $UNC^{AR} \times ISE$ | | 1.170*** | | | | |
| | | (0.308) | | | | |
| $UNC^{S}$ | | | -0.006** | -0.016*** | | |
| | | | (0.003) | (0.004) | | |
| $UNC^{S} \times ISE$ | | | | 0.083*** | | |
| | | | | (0.025) | | |
| Cycl. shock ($BK$) | | | | | -0.101*** | -0.257*** |
| | | | | | (0.032) | (0.043) |
| Cycl. shock ($BK$) $\times$ $ISE$ | | | | | | 1.380*** |
| | | | | | | (0.298) |
| Other CONTROLS | Y | Y | Y | Y | Y | Y |
| Constant | 2.646*** | 2.623*** | 1.948*** | 1.908*** | 2.000*** | 2.081*** |
| | (0.312) | (0.311) | (0.272) | (0.272) | (0.326) | (0.305) |
| Threshold Index | | 0.29 | | 0.19 | | 0.19 |
| Pseudo $R^2$ | 0.90 | 0.90 | 0.89 | 0.90 | 0.91 | 0.91 |
| Observations | 22,068 | 22,068 | 23,847 | 23,847 | 21,684 | 21,684 |

## Table 7: **By Firm Size**

| | Grants by Filing Date | | | |
|---|---|---|---|---|
| | Below Median | Below Median | Above Median | Above Median |
| | (1) | (2) | (3) | (4) |
| Ln R&D Flow | 0.020 | 0.026 | 0.293*** | 0.291*** |
| | (0.040) | (0.039) | (0.030) | (0.029) |
| Ln Sales | 0.138** | 0.170*** | 0.123** | 0.130*** |
| | (0.055) | (0.055) | (0.050) | (0.050) |
| *UNC* | -0.452*** | -1.197*** | -0.110** | -0.409*** |
| | (0.135) | (0.251) | (0.052) | (0.105) |
| *UNC* × *ISE* | | 4.161*** | | 2.206*** |
| | | (0.990) | | (0.514) |
| Other CONTROLS | Y | Y | Y | Y |
| Constant | -0.259 | -0.154 | 2.188*** | 2.223*** |
| | (0.615) | (0.595) | (0.337) | (0.327) |
| Threshold Index | | 0.29 | | 0.19 |
| Pseudo $R^2$ | 0.80 | 0.80 | 0.89 | 0.89 |
| Observations | 11,615 | 11,615 | 11,546 | 11,546 |

## Table 8: **By Select Sectors**

| | Grants by Filing Date | | | |
|---|---|---|---|---|
| | INFO TECH | INFO TECH | BIO-PHARM | BIO-PHARM |
| | (1) | (2) | (3) | (4) |
| Ln R&D Flow | 0.280*** | 0.276*** | 0.363*** | 0.368*** |
| | (0.039) | (0.040) | (0.063) | (0.063) |
| Ln Sales | 0.069 | 0.084 | 0.015 | 0.019 |
| | (0.054) | (0.054) | (0.093) | (0.093) |
| *UNC* | -0.192*** | -0.914*** | -0.622*** | -0.739*** |
| | (0.056) | (0.119) | (0.147) | (0.188) |
| *UNC* × *ISE* | | 4.056*** | | 8.761 |
| | | (0.563) | | (5.639) |
| Other CONTROLS | Y | Y | Y | Y |
| Constant | 1.040*** | 1.124*** | 4.223*** | 4.187*** |
| | (0.362) | (0.349) | (0.582) | (0.577) |
| Threshold Index | | 0.22 | | n/a |
| Pseudo $R^2$ | 0.90 | 0.90 | 0.77 | 0.77 |
| Observations | 8,999 | 8,999 | 2,962 | 2,962 |

Table 9: Quasi-simulation: Period 2017 - 2021

Shock to Revenue = 10% of Std Deviation

| Companies | Revenue Change % | Uncertain. Change % | Exposure Index | Total New Grants | Indirect New Grants | Original Grants |
|---|---|---|---|---|---|---|
| Bristol-Myers | 3.40 | -3.32 | 0.009 | 644 | 506 | 279 |
| Gilead | 4.43 | 1.57 | 0.010 | 126 | 72 | 108 |
| Microsoft | 3.52 | 0.56 | 0.299 | 5837 | 4031 | 3747 |
| Qualcomm | 4.31 | -0.74 | 0.314 | 4945 | 3148 | 3729 |

Indirect effect measures revenue change on patenting via uncertainty
Total effect includes direct effect of revenue change on patenting

## Formula:

$P_i^{New} = P_i(1 + g_r)$

where the percentage growth in patent grants for firm i is:

$g_r = \mathbf{0.118} \times (\%\Delta \text{ in Revenues}) + (\mathbf{2.478} ISE_i - \mathbf{0.501}) \times (\%\Delta \text{ in Uncertainty})$

*Coefficient estimates (in* **bold***) from Table 5, col. 2*

# Recap

- Results consistently show $\beta_1 < 0$ and $\beta_2 > 0$. That is, uncertainty has a **negative** association with patenting, but above a threshold level of secondary patent market exposure, uncertainty has a **positive** association with it. Threshold varies by firm size and sector.
- A priori, uncertainty has diverse potential effects, which can be associated with higher revenues or lower revenues (namely, the **volatility**).
- Positive implication: study explains the **resilience** of some firm patenting under uncertainty.
- (Potential) Normative implication: expand **secondary markets for patents**. Improve their efficiency. Formalize like equity markets.
- Future research could investigate **other factors** that make innovation resilient: STEM education and training, institutional support (IP offices), labor market for R&D personnel, organizational and managerial competence, public policy support, and networks (regional and international).
- Future work could also try to **replicate** the study for other countries or regions (if secondary patent markets exist and data on sales are available)