An Optimizing Model of R&D, Spillovers, and Intellectual Property Rights*

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Abstract

This paper develops a microeconomic model of private R&D and studies the effects of intellectual property rights (IPR) on equilibrium R&D. IPR has two opposing effects: on the one hand it enables a firm to better appropriate the returns to R&D; on the other hand, it has the potential to impede the flow of R&D knowledge spillovers among firms. Spillovers may be impeded if it is necessary for users to practice R&D in order to benefit from R&D externalities.

Keywords: R&D, Intellectual Property Rights, Spillovers

JEL Classification: O31, O34, L16

Ar-Ge, Yayılımlar ve Fikri Mülkiyet Haklarının Optimize Edici Bir Modeli

Özet

Bu makalede, özel Ar-Ge’nin mikroekonomik modeli geliştirilmiş ve fikri mülkiyet haklarının (FMH) Ar-Ge üzerindeki etkileri araştırılmıştır. FMH’nin iki karşılıt etkisi vardır: bir yandan bir firmanın, Ar-Ge’ye getirilerini daha iyi bir şekilde sahiplenmesini sağlar; öte yandan firmalar arasında Ar-Ge bilgilerinin yayılımlarının akışına engellemesini potansiyelinde sahiptir. Ar-Ge dışsallıklarından yararlanmak için kullanıcıların Ar-Ge çalışmalarında bulunmaları gerektiğini, yayılımlar engellenebilir.

Anahtar Kelimeler: Ar-Ge, Fikri Mülkiyet Hakları, Taşmalar

JEL Sınıflandırması: O31, O34, L16

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1. Introduction

The focus of this paper is the relationship between private R&D and intellectual property rights. There is a common belief that market failures exist in R&D, due to the public good characteristics of knowledge and externalities associated with R&D activities. One solution is to have public sector funding and/or performance of R&D. Another is to foster private markets for R&D by creating private ownership or property rights over the products of R&D. The focus in previous work has been primarily on the relationship between public R&D and private R&D.¹

There are different views as to the ultimate effect of IPRs; namely, whether they enhance R&D capital accumulation or restrict R&D knowledge diffusion. The proponents of stronger IPRs generally emphasize the potential effect on stimulating R&D. The opponents emphasize the costs of stronger IPRs, which include limiting knowledge diffusion and frustrating the R&D of those who do not own the property right (for example, the R&D of rivals and follow-on inventors). Thus the impact of IPRs on R&D is to date an unresolved issue.

This paper develops a simple dynamic equilibrium model of R&D that captures these different channels.

The next section briefly reviews prior work. Section 3 analyzes the model, and section 4 concludes.

2. Selected Previous Work

The impact of intellectual property rights on R&D remains the subject of theoretical, empirical, and policy controversy. Research on the determinants of private R&D has thus far been dominated by discussions of the role of public R&D, subsidies, tax credits, market size, firm size, corporate profits, and concentration ratios, among others.²

¹ See, for example, David and Hall (2000) for a survey and analysis.
² See David, Hall, and Toole (2000) for a survey.
Conclusions about the welfare and efficiency effects of stronger intellectual property rights are quite mixed. For example, Takalo and Kanniainen (2000) find that a strengthening of patent rights can delay the introduction of a new technology to the market (i.e. raises the value of the innovator to wait); Futagami and Iwaisako (2007) show that social welfare can be reduced if longer patents introduce much static inefficiency, whereby the proportion of competitively produced goods is reduced; and Bessen and Maskin (2009) develop a model of sequential and complementary innovation in which patent protection reduces innovation and social welfare. Chu et al. (2012) show that patents can differentially affect vertical innovation and horizontal innovation, and thus affect economic growth in complex ways. Bournakis et al. (2018) provide a rich analysis of how knowledge spillovers can drive productivity, and how stronger IPR helps economies absorb them.

Landes and Posner (2003) show that there are tradeoffs involving trademark rights and copyrights as well. Essentially trademark protection encourages economic efficiency by reducing the search costs of consumers. Furthermore, firms or intellectual property owners in turn invest in promotional expenditures to attract consumers and expenditures to maintain the quality of their products or services. Without being able to link their investments and products to their trademark, they would have less incentive to invest in those quality-promoting investments. There are occasions, however, where trademark protection can be too broad (e.g. a name or symbol becomes generic) and would increase the cost of business for rival firms such that economic efficiency is harmed in the aggregate. Copyrights over original and derivative works also stimulate creativity by increasing the odds of appropriating the benefits of the creations. But situations may also exist where stronger copyrights are adverse to economic efficiency -namely where the rights reduce the incentive of rivals to create, or the owner’s incentive to produce future new creations. Each creator is part of an intertemporal chain of creators. Thus stronger protection on expressions affect subsequent generations of creators who themselves would like to build on previous works.

In much of this work, however, an optimizing model of R&D is not employed, but rather ad hoc models of R&D behavior are utilized or imposed, even if the larger model in which an R&D sector is embedded has a dynamic general equilibrium setting. The next section develops a simple tractable model of R&D and intellectual property rights.

3. Model and Analysis

The following is an optimizing model of the demand for and supply of R&D capital. On the buyer’s side, there exists a single aggregate representative firm that behaves competitively which, in order to produce output, purchases R&D inputs from private producers to be used along with other factors of production: labor

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3 For earlier, non-optimizing equilibrium models of R&D behavior, see Lichtenberg (1987) and Howe and McFetridge (1976). The underlying basis is similar: investment in R&D proceeds to the point at which the marginal rate of return to R&D equals the marginal cost of funds.
and capital. Its output also depends upon
the stock of public (government-financed)
R&D capital. On the supply side, there
are many firms producing R&D output
for the demand side of the market. A
single representative firm could have been
posed, with no qualitative change in
results (and indeed symmetric identical
suppliers will be assumed shortly), but this
modelling approach allows us to see how
inter-firm R&D spillovers can occur. The
producers of R&D output solve an (almost)
standard profit maximization problem,
choosing their level of output to maximize
profits; however, the cost of producing
R&D output depends not only on the level
(or quantity) of R&D output, but also on
the stock of spillover R&D capital available
to the firm. This feature captures the idea
that there exist intertemporal knowledge
spillovers. The marginal cost of producing
R&D is lowered by a larger stock of past
R&D available in the economy. The more
knowledge there is, the less costly it is
(holding other factors constant) to produce
a given increment of knowledge. This helps
to capture the “standing on the shoulders of
giants” effect. Indeed the firm benefits not
only from having done more R&D in the
past but also from other firms having done
more R&D in the past. Thus intertemporal
spillovers derive from both own R&D and
spillover R&D. How easily available or
accessible other firms’ R&D knowledge is
for the use of a firm depends, among other
things, on the degree of substitutability
between R&D performed by different
firms and on whether or not there are
technical and legal barriers affecting the
transmission of knowledge from one firm
to another (or from one sector to another,
or even from one country to another).

This brings us to the question of how
intellectual property rights play a role in
the generation of R&D output. On the
one hand, private producers can better
appropriate the returns to their R&D
efforts. The R&D is sold to the prospective
users of R&D capital. With imperfect
appropriability, the producers only capture
a fraction of the potential sales to the users
(or buyers). Imitators capture the remaining
fraction of sales. Either way, imperfect IPR
is modelled as reducing the revenues to the
supplier. On the other hand, IPRs may be
one type of legal barrier affecting inter-firm
spillovers. To the extent that intellectual
property protections reduce knowledge
diffusion, the degree of spillovers will be
smaller, making it more difficult for firms
to acquire spillover knowledge or enjoy the
advantages of standing on the shoulders of
previous researchers.

Of course, there is one difficulty with
this argument about IPRs inhibiting
knowledge diffusion. What IPRs actually
are intended to do is to exclude non-rights
holders from practicing or exploiting the
new ideas (inventions, creations, and so
forth). But IPRs do not preclude others from
benefitting from the knowledge per se, as
long as they apply the knowledge in some
alternative way or produce a differentiated
idea, concept, or expression. Indeed, an
objective of having IPRs (particularly in
the case of patent rights) is to encourage
the rights-owner to disclose or reveal
the underlying technology (sufficiently to enable persons reasonably skilled in the practice to replicate or duplicate the new idea). Thus, often the (temporary) intellectual property protection is given explicitly in exchange for the disclosure of new knowledge. In the case of patents, the disclosure is provided in the patent application itself and is available to the general public in print form, online, or other media. Thus, far from concealing knowledge, the objective of IPR laws is actually to enhance it. One therefore has to distinguish between technology diffusion (of intellectual products or services in the marketplace) from knowledge diffusion.

Still, a case could be made that while knowledge diffusion itself is not affected, the manner in which knowledge can be used in research activity is affected by the laws and protections granted to others. The essence of spillovers is that research done by others has some beneficial effect (i.e. some positive externalities) on an agent. But the benefits may largely be derived only if the agent is able to exploit it in some fashion or other, and that certain laws may prohibit that exploitation. In this conceptualization, the enjoyment of spillovers requires some learning by doing or learning by producing. Tighter intellectual property protection may restrict an agent's ability to derive spillover benefits precisely because IPRs limit the manner in which non-rights holders may use or apply any proprietary knowledge. Thus, as long as learning by doing is an important means by which research spillovers can take place, then IPRs may weaken the beneficial (spillover) effects of knowledge diffusion. Under this view, knowledge diffusion or disclosure occurs but the economic impacts of that diffusion can be limited.

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4 Another problematic argument is that IPRs (particularly patent rights) impede knowledge diffusion because private agents will acquire proprietary rights over basic knowledge, which is vital to furthering knowledge. The argument is misplaced because technically patent rights are not granted to basic scientific and mathematical knowledge. Proprietary rights can be obtained for applied work, not theoretical work. This is not to say that in practice proprietary rights have not been given to what many would consider “theoretical” discoveries (for example, “expressed sequence tags” in genome research), but those patent grants would be considered granted in “error”, and grounds for invalidation would exist.

5 This is not to say that the knowledge diffusion is limited to spillovers of technological knowledge between firms. There may, for instance, be useful spillovers for academic research. Moreover, even if no technological spillovers occur, there may be some valuable signals. Firms are provided with information on which technological areas have been covered (or solved) and which are open for discovery or exploitation (or further research). Intellectual property disclosure also has other functions. The patent is not only a scientific document but also a legal document. It shows what rights and intellectual claims the holder has. It is a document for purposes of legal enforcement. Hence, another kind of knowledge disclosure is knowledge about the market structure, its organization, niche, and positioning of competitors, if any.
An Optimizing Model of R&D, Spillovers, and Intellectual Property Rights

The following is a list of notation to be used:

- **IPR** Level of Intellectual Property Right
- **R_p** Stock of Private R&D Capital
- **R_g** Stock of Public R&D Capital
- **I_p** Investment (Flow) in Private R&D Capital (and Measure of Private R&D Output)
- **I_g** Investment (Flow) in Public R&D Capital
- **Y** Output
- **V** Value of Firm
- **π** Profits
- **i** Investment in Private R&D Capital by the ith-firm
- **r_i** Stock of Private R&D Capital of the ith-firm
- **R_i** Stock of Spillover Private R&D Capital of the ith firm
- **N** Number of R&D-Producing Firms
- **c** Cost of Producing Private R&D Output
- **P_R** Price of Private R&D Output
- **δ** Geometric Depreciation Rate of R&D Capital
- **ρ** Real Interest Rate
- **ω** Degree of Substitutability between Own R&D and Spillover R&D
- **θ** Degree of Appropriability or Market Share of Intellectual Property Rights Holder
- **ψ** Degree of R&D Spillovers derived by firms

In what follows, the optimizing demand for R&D will first be characterized, followed by the optimizing supply of R&D. The two equations are then solved to yield the steady-state stock of R&D capital. On the demand side, the representative aggregate firm maximize the following functional:

\[
\max_{R_p, R_g} V = \int_{t}^{\infty} \left[ Y(R_p, R_g, ...) - p_R I_p \right] e^{\lambda s} ds
\]

subject to:

\[
(2) \quad R_p = I_p - \delta R_p
\]

where output Y is a function of the stocks of private and public R&D capital. (For the moment, we suppress the other factors of production, such as physical capital and labor.) The following first and second partial derivatives of the production function are assumed to ensure positive marginal products and diminishing returns: \( Y_1 > 0, Y_2 > 0, Y_{11} < 0, \) and \( Y_{22} < 0. \)

Public R&D investment, \( I_g \), is treated as exogenous here, and public R&D capital evolves as follows:

\[
(3) \quad R_g = I_g - \delta R_g
\]

The necessary condition for value maximization is:

\[
(4) \quad Y_1 = (\rho + \delta) p_R - p_R^{i_k}
\]

This is the standard optimality condition where the LHS is the marginal product of R&D capital \( (\delta Y / \delta R_p) \) and the RHS the user cost of R&D capital.
On the supply side, assume N producers of R&D output. The ith firm (where \( i = 1, \ldots, N \)) chooses its level of R&D output, \( t_i \), to maximize the present discounted flow of profits:

\[
(5) \quad \max_{t_i} \Pi_i = \int_{0}^{\infty} \left[ \theta p_i t_i - c(t_i, R^*_i) \right] e^{-\delta s} ds
\]

subject to

\[
(6) \quad t_i^{\alpha} = t_i - \delta t_i
\]

where \( p_i t \) represents firm revenues, of which a fraction \( \theta \) is actually appropriated by the firm due to imitation. It is assumed that \( \theta = \theta (\text{IPR}) \), where the partial derivative \( \theta' > 0 \) and \( 0 \leq \theta \leq 1 \). In the absence of imitation, \( \theta = 1 \); under perfect imitation, \( \theta = 0 \).

The interpretation here is that \( \theta \) is the market share of the intellectual property rights holder and \( 1 - \theta \) the share of imitators. Both the rightful owner and imitators compete and charge the same price \( p_R \) to the user of R&D and produce identical outputs. Of course, equilibrium \( p_R \) could be higher (or lower) the fewer (or more) imitators there are. At that price, if \( I_p \) denotes the total R&D output demanded, the composition of output produced between the rights holders and imitators are as follows:

\[
I_p = \theta I_p + (1 - \theta) I_p, \text{where } I_p = \sum t_i, \text{for } i = 1, \ldots, N
\]

In (5), \( c(t_i, R^*_i) \) is the cost function for R&D output. The cost of producing R&D output depends positively on the quantity of output produced, \( t \), and negatively on the stock of spillover R&D knowledge, \( R^* \), where

\[
(7) \quad R^*_i = r_i + \Psi \sum_{j \neq i} w_i r_j
\]

and

\[
(8) \quad R_p = \sum_{i=1}^{N} r_i
\]

That is, the stock of R&D generates intertemporal (including inter-firm) externalities. Firm's investments (or experiences) in R&D generate some learning by doing externalities (which affect the future marginal cost of R&D production). It is assumed that \( c_1 > 0, c_{11} > 0, c_2 < 0, c_{22} > 0, \) and \( c_{12} < 0 \). Thus, an increase in the stock of spillovers reduces the marginal cost of R&D production but at a diminishing rate.

In (7), \( R^*_i \) is the spillover R&D available to the ith firm. It is composed of the firm's own past R&D stock (that is, the knowledge capital that it created) and of other firms' R&D stock. This may include the R&D of firms in other sectors (or other countries). \( \omega \) measures the degree of substitutability between one firm's R&D and that of another. It may also be referred...
to as the measure of technological similarity between sectors. It is likely that the R&D of other firms would be less appropriate to the research activity of a given firm. In some cases, no usefulness of research might be expected between firms in diverse sectors, yet some spillovers have been generated (e.g. between semiconductor research and quartz watch research, or heart surgery research and automotive research).

As discussed earlier, R&D spillovers from other firms to the ith firm may be "blocked" by the intellectual property rights of other firms which prohibit the ith firm from utilizing the knowledge of others, thereby limiting its ability to capture R&D spillovers. ψ measures the extent to which external R&D “spills” in, where it is assumed that ψ(IPR), and the partial derivative ψ’ < 0 and 0 ≤ ψ ≤ 1. Where IPRs do not prohibit spillovers at all, ψ = 1; and where IPRs completely shut off spillovers, ψ = 0. Note in (7) that the variable ψ does not affect the ith firm’s ability to enjoy spillovers from its own past R&D, r_i.

Before characterizing the optimality conditions for the supplier (i.e. the solution to (5)), it is worth going over some special cases:

**Case 1: No Intertemporal Spillovers**

In this case, the cost function for producing R&D output is just c = c(t_i), and the necessary condition for profit maximization is the standard condition:

$$\theta p_R = c'(t_i)$$

whereby the level of R&D output, t_i, is chosen at which the price of R&D output, adjusted for the degree of appropriability of revenues, equals the marginal cost of production. Inverting the necessary condition gives t_i = t_i(\theta p_R). Since all firms face the same price and if they also have identical cost functions, t_i = t; that is, they will produce the same amount of R&D output. Hence aggregate R&D output (or aggregate investment from the point of view of the demand side) is:

$$I_P = I_P(\theta p_R) = N_\alpha(\theta p_R)$$

The supply of R&D is positively related to the price of R&D capital, which in steady-state is (using equation (4)):

$$p_R = Y_i/(\rho + \delta)$$

Thus, in steady-state, I_P = I_P(\theta Y_i/(\rho + \delta)); that is, it is a function of the discounted marginal productivity of private R&D capital, adjusted for the degree of appropriability. In this case, an increase in the stock of private R&D capital would depress investment due to the diminishing marginal productivity of R&D capital. And tighter intellectual property protection, to the extent that it raises the degree of appropriability of R&D revenues, θ, would stimulate R&D investment. In the next two cases, these two predictions are ambiguous.

**Case 2: Small Firm Case – Exogenous Spillovers**

In this case, in definition (7), r_i is considered sufficiently small relative to the stocks of R&D of other firms that the ith firm treats the stock of (economy-wide)
spillovers $R^*$ as given. The firm would not take into account the contribution of its R&D output to future cost reductions. The cost of producing R&D output decreases with the stock of R&D spillovers (or with society's stock of R&D), but the firm's own output constitutes a negligible share. The effect of this (compared to the next case) is that the firm's problem remains "static" as in Case 1 above; that is, the firm chooses output to maximize instantaneous profits. The necessary condition for profit maximization is:

$$\theta p_R = c'(r_i, R^*)$$

In the special case where $\psi = \omega = 1, R^*$ simply coincides with the aggregate stock of private R&D capital, $R_p$. Combining the above condition with the optimizing condition from the demand side (4), aggregate R&D investment, $I_p$, will be found in steady-state equilibrium to have an ambiguous relationship with the aggregate stock of private R&D and with the level of IPR. A higher stock of aggregate private R&D, on the one hand, reduces the cost of producing an additional unit of R&D output; on the other hand it reduces the marginal productivity of R&D capital (and reduces the market's demand for R&D output). A higher IPR level increases the appropriability of R&D investment on the one hand and may (on the other hand) reduce the transmission of spillovers among firms. Thus the overall impact on equilibrium R&D is ambiguous. This can help explain why intellectual property systems are rather complex and their effects not fully predictable. As described here, the effects are conditional on how IPR affects the transmission of knowledge spillovers.

Case 3: Large Firm Case – Endogenous Spillovers

In this case, we allow for the possibility that the R&D supplying firm is large enough to realize that its own production of R&D helps to reduce its future marginal cost of R&D production. The steady-state comparative statics effects of a change in the aggregate stock of private R&D and in the level of IPR are the same (as in Case 2). Indeed, one purpose of comparing cases 2 and 3 is to show that the ambiguous effects of IPR and the stock of private R&D capital on R&D investment do not depend on the assumption of a small vs. large firm. However, both the dynamics and the steady-state equilibrium levels of R&D are affected by the nature of the firm and how it perceives its stock of R&D vis-a-vis that of others.

For the $i$th firm, the solution to maximizing (5) subject to (6), via the choice of $r$ and $r^K$ is:

$$\phi^c (\psi, R^*) = c_1 + \theta p_R$$

Its own R&D also contributes to the stock of R&D spillovers of other firms (as theirs contribute to this firm’s spillover R&D stock). The firm, however, does not take into account the benefits other firms derive when choosing how much R&D to produce. Overall, the equilibrium amount of R&D should be lower than it would otherwise be. It would be a useful extension to develop a model and data set that would help determine the extent to which underinvestment in R&D occur (vis-a-vis a social planner outcome).
where \( c_2 = \frac{\partial c}{\partial R_1} \) is the marginal reduction in cost due to R&D spillovers, and \( c_1 = \frac{\partial c}{\partial t} \) is the marginal cost of producing R&D. \( z \) is the excess of the marginal cost of producing R&D over the marginal benefit of producing R&D. \( z > 0 \) as long as spillover research and development has a beneficial “negative” effect on the marginal cost of producing R&D. The firm is led to produce beyond the point where the marginal cost equals the marginal benefit of producing R&D. Figure 1 illustrates the optimal R&D output and the size of \( z \). Figure 2 shows how the equilibrium R&D output increases with a larger stock of spillovers.

Figure 1: Optimal R&D

Figure 2: Impact of Increased Knowledge Spillovers
Reduced-Form Impacts

With symmetry, \( t_1 = t, r_1 = r \), and thus \( R' = (1 + \psi(N - 1))r \). Now, in steady-state, using equations (2)-(4), (6), and (8)-(10), we obtain:

\[
(11) \ c_1 \left( \frac{\delta R_p}{N}, (1 + \psi(N - 1)) \frac{R_p}{N} \right) + \frac{c_2 \left( \frac{\delta R_p}{N}, (1 + \psi(N - 1)) \frac{R_p}{N} \right)}{\rho + \delta} = \frac{\theta Y_1(R_p, R_g)}{\rho + \delta}
\]

Totally differentiating (11) with respect to \( R_p, R_g \), and IPR yields:

\[
(12) \ dR_p = \Omega_g \ dR_g + \Omega_i \ dIPR
\]

where

\[
(12a) \ \Omega_g = \frac{\theta' Y_{12}}{\rho + \delta} + \delta \left( c_1 + \frac{c_2}{\rho + \delta} \right) > 0
\]

\[
(12b) \ \Omega_i = \frac{\theta' Y_{1}}{\rho + \delta} - (N - 1) \psi' r \left( c_{12} + \frac{c_{22}}{\rho + \delta} \right)
\]

\[
(12c) \ \Omega = \frac{1}{N} \left[ \delta \left( c_{11} + \frac{c_{21}}{\rho + \delta} \right) + (1 + (N - 1) \psi) \left( c_{12} + \frac{c_{22}}{\rho + \delta} \right) \right] - \frac{\theta Y_{11}}{\rho + \delta} > 0
\]

Thus \( \Omega = dR_p/dIPR \) measures the comparative statics effect of a change in the IPR level on the steady-state stock of private R&D. The coefficient indicates what the effect depends on. The first term is positive and the second term negative. It is assumed that in absolute value \( |c_{12}| < |c_{22}| \), implying that, across steady-states, the stock of spillover R&D reduces the marginal cost of producing R&D (even after allowing for the fact that the steady-state level of R&D produced will be higher). The first term measures the impact of IPR on the appropriability of R&D returns; the second measures the extent to which IPR “limits” (if at all) spillovers from occurring. The second term would vanish if \( \phi = 0 \) (meaning that external R&D is not appropriate to the firm). The effect of having more firms (i.e., having larger \( N \) across steady-states) is also ambiguous: more firms could mean more potential spillovers or more spillovers forgone due to any restrictions imposed by IPRs. Note that in (12c), \( \Omega \) is positive as long as the overall steady-state cost of producing R&D rises with the stock of private R&D even after netting out the effect of intertemporal and interfirm spillovers. Otherwise, the equilibrium stock of private R&D would approach infinity.
The sign of $\Omega_{\varepsilon}$ is positive as long as the cross partial derivative $Y_{12}$ is positive; that is, public R&D raises the marginal productivity of private R&D and vice versa.

A few words on the dynamics. The dynamics of the system are governed largely by equations (4), (9), and (2). Combining (4) and (9) yields an equation of motion for the marginal cost of R&D output, $c_1$. Along the equation where $c_1' = 0$, the marginal cost and the stock of private R&D vary inversely. Holding the marginal cost constant, across steady-states, a higher stock of private R&D is associated with lower marginal returns to R&D capital ($Y_1$) and lower marginal returns to spillover R&D, since spillover R&D reduces the marginal cost of R&D production at a decreasing rate, given that $c_{22} > 0$. This would put pressure on the marginal cost of R&D production to increase. Thus, in order to keep $c_1$ constant, a lower marginal cost must be associated with a higher stock of private R&D.

Now, R&D investment is a function of the marginal benefit of R&D, namely $\theta p_R$, but this in turn equals the marginal cost of R&D, $c_1$, in equilibrium. Hence investment in R&D is a function of $c_1$; more specifically a positive function. As Figure 3 indicates, marginal cost $c_1$ and investment $I_p$ co-vary positively. Thus the equation of motion for the stock of R&D (equation (2)) is indirectly a function of $c_1$. As shown in Figure 4, the equation for $R_0 = 0$ is positively sloped. A greater stock of private R&D involves greater depreciation, and therefore requires a greater steady-state flow of investment. A higher steady-state investment in R&D is associated with a higher steady-state marginal cost of producing R&D. Figure 4 indicates also the saddlepath trajectory towards steady-state. Along this path, as the marginal cost falls (rises), the stock of private R&D rises (falls).
4. Conclusion

As indicated earlier, public support for R&D and intellectual property protection are both solutions to the appropriability problem in R&D knowledge accumulation (though not the only solutions). In the absence of public R&D or intellectual property rights, underinvestment (or no investment) in R&D is argued to result. Thus, public R&D and IPRs should at least raise R&D levels above what they would be without public intervention or proprietary rights.

However, this paper identified two opposing effects of strong IPRs: the appropriability effect and spillover reduction effect of tighter IPRs. The first effect is well-documented and studied in the existing literature. The second effect has not been formally analyzed thus far in the literature, and to that end, this paper fills a gap. The premise of the model in this paper is that inter-firm knowledge spillovers are not automatic. Firms can better exploit knowledge spillovers by engaging in research activities and learning-by-doing.\textsuperscript{10} To the extent that intellectual property protection –be it patent rights, trademark rights, or copyrights– restricts the ability of firms to employ new methods of production or new technological outputs, fewer knowledge spillovers are transmitted across organizations or over time. Thus, while IPRs increase the incentive to conduct own-R&D, firms are limited from enjoying spillovers from R&D conducted by other firms. This paper showed that in the absence of intertemporal spillovers, stronger IPRs stimulate firm R&D unambiguously (recall Case 1). But once such spillovers are allowed –exogenously or endogenously– the impact of IPR on R&D is positive as long as any spillover reduction effect is not too great (see Cases 2 and 3).

The results have both empirical and policy significance. Empirically, studies

\textsuperscript{10} Kealey and Richards (2014) also develop the point that scientific knowledge is not automatically accessible.
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should control for the possible spillover reduction effects of IPR.\textsuperscript{11} Policy-wise, lawmakers and regulators intending to promote innovation via stronger intellectual property protection should take into consideration the general equilibrium effects.\textsuperscript{12} Stronger protection may not stimulate the innovation of a firm or industry if it impedes the flow of knowledge from other firms, industries, or countries.


\textsuperscript{12} See Baker et al. (2017) for a review of the policy debates on IPR.

\textbf{Bibliography}


