Math Pre-test Answer Key and Review Guide

This document gives the answers to the Math Pre-test for Microeconomics that is found on Professor Hansen's homepage. Please report broken links to Professor Hansen via e-mail: mhansen at american.edu.

Questions 1-3 cover fractions and percentages.

1. $2/8 = 25\%$

2. $(75\text{-}60)/60 = 15/60 = 25\%$

3. $\frac{(5-3)/2}{(7-6/2}$ = ? Numerator equals 2/2=1. Denominator equals ½. Simplifies to 2. Can also solve by rearranging as $\frac{5-3}{7-6} \times \frac{2}{2} = \frac{2}{1} \times 1 = 2.$

    We will use fractions and percentages primarily for application called "elasticity". Elasticity is introduced immediately after "supply and demand" (Mankiw Ch. 5-6), used extensively in "welfare analysis" (Mankiw Ch. 7-9), and revisited in the "theory of the firm" and "factors of production" (Mankiw Ch. 16-19).

    Review converting fractions to decimals and percentages in the Khan Academy tutorial on the Graphical Slope of a Line. Review simplifying fractions Khan Academy tutorial on Fractions in Lowest Terms. Review simplifying complex fractions in the tutorial on Adding Complex Fractions by Charlie Lindelof.

Questions 4-6 cover graphs in the coordinate plane.

4. Point C is (2, 5). 2 is the *x*-coordinate; 5 is the *y*-coordinate.

5. A line through points A and D has slope -3/3 = -1. The absolute value is 1.

6. A line through points A and D has an *x*-intercept of 6.

    We will use points on the coordinate plane and lines in every chapter beginning with Chapter 2.

    Review plotting points using the Khan Academy video called Plot Ordered Pairs. Review graphing lines using the Khan Academy video called Plotting (x,y) Relationships. Review finding the equation of a line through two points by watching the Khan Academy tutorial called Algebra: Equation of a Line.

Question 7 covers identifying shapes and finding their areas.

7. The quadrilateral (this one is a trapezoid) ABCD is composed of a rectangle and a triangle. To see this, add a point E to the graph at (3, 5). The rectangle is ABDE. Its area is 4*2=8. The triangle is CDE. Its area is ½*1*2.

    We will use areas of rectangles, triangles, and quadrilaterals to find values of revenue, taxes, and various concepts in "welfare" economics. The first occurrence is in Chapter 5.

    Review calculating areas of polygons by watching the Khan Academy video on Perimeter and Area of a Non-Standard Polygon.

Questions 8-9 cover solving equations with one unknown.

8. If $\frac{x}{4} = \frac{15}{20}$, then x=3.
9. If $3x = 12 - x$, then x=3.

    We will use basic algebra in every part of the course to do comparisons and to find missing values. The underlying equations are generally definitions of economic terms. We are told some parts of the equation and must solve for an unknown. A classic example is in a discussion of taxes: we are asked to recommend how large a "sin" tax is necessary to reach some goal for reducing the number of cigarettes smoked.

    Review the basics of solving for a single unknown by watching the Khan Academy tutorial Solving Equations 1. Review the intuition of solving equations by viewing the Khan Academy Video One-Step Equation Intuition. There are lots of additional examples on the Khan Academy Algebra page.

10. If $y = 10 - x$, and x=3 then y=7.

11. If $y = 10 - x + 2z$, and x=3 and z=2, then y=10-3+2*2=11.

We will evaluate expressions regularly beginning with the topics of "supply and demand" (Mankiw Ch. 4). It will be essential when we think about the cost of making something and the profit firms make (Mankiw Chs. 12-16).

Review evaluating expressions with one variable by viewing the Khan Academy video on Evaluating Functions. Review evaluating expressions with two variables in the Khan Academy video Expressions with Two Variables. A very important related skill is evaluating an expression when it is represented by a graph; practice using the Khan Academy interactive tutorial Views of a Function.

Questions 12 covers inverting equations.

12. If $y = 10 + \frac{1}{2}x$, then $x = 2y$ - 20

We will use inverse functions to work demand and supply problems. Review inverse functions by watching the Khan Academy video Introduction of Function Inverses.

Questions 13-15 cover the basics of graphs of lines.

13. The graph of *y*=10-2*x* has a *y*-intercept of 10 and a slope of -2.

14. The graph of *y*=10-2*x* has an *x*-intercept of 5.

15. Combine the constants to find the *y*-intercept: 10+4=14.

We will use graphs of lines to illustrate simple versions of nearly every concept.

Review the simplest way to graph lines by watching the Khan Academy video Graphing a Basic Function. Also watch the Khan Academy tutorial on X and Y Intercepts.

Question 16 covers solving a simple system of linear equations.

16. To find the intersection of the lines $\boxed{\begin{array}{l} y = 10 - 2x \\ y = 3x \end{array}}$, substitute $3x$ for y in the first equation:

$3x = 10 - 2x$. $5x = 10$ so $x = 2$.

This is an essential tool for understanding demand and supply (Mankiw Chs. 4-9). Other intersections will be important to the theory of the firm (Mankiw Chs. 13-16).

Review the meaning of a graphic solution to a system of equations by viewing the Khan Academy tutorial Solving Systems by Graphing 2. Get Simple Elimination Practice via Khan Academy as well.

Questions 17-18 cover approximating the slope of a curve.

17. When $x=8$, $y=8$. When $x=9$, $y=10$. Y increased by 2.

18. When $x=1$, $y=3$. When $x=2$, $y=4$. Y increased by 1.

We will need to find and interpret the slopes of curves (and changes in the slopes of curves) when we work on "production possibilities" (Mankiw Ch. 2) and especially when we consider a firm's production and costs (Mankiw Chs. 13-14).

Review approximating slopes of curves by using the Interactive mathematics chapter on The Slope of a Tangent to a Curve. Note the vocabulary: the slope of a line that is "tangent" to the curve is the exact slope of the curve at a point. This course does not require calculus, but it is useful to review the intuition of the first derivative by viewing the Calculus: Derivatives 1 tutorial by Khan Academy.

Questions 19-20 cover identifying the types of functions by their equations.

19. The equation $y = 10 - x$ describes a line. The slope, which is equal to -1, is the same everywhere on the function. There are no exponents on either x or y.

20. The equation $y = (10-x) \, x$ is NOT a line. Rewrite it: $y = 10x + x^2$. It is a parabola.

We'll need to be able to go between equations and graphs when we are working on "demand and supply", revenues, profits, and costs.

Review using the Graphs of Quadratic Functions from the Khan Academy.

21. The slope of the line segment between A and C is -1/2. The slope of the segment between C and D is -2. |-2| > | -1/2|

22. This function is getting steeper as x increases, so its slope is getting bigger. You can see this by comparing slopes of line segments on $1<x<2$ and $7<x<8$ or by comparing the slope of the tangent line at different x values.

23. This function is getting flatter as z increases, so its slope is getting smaller. Again, comparing slopes of line segments on $1<x<2$ and $7<x<8$ or by comparing the slope of the tangent line at different x values.

24. This is a cubic function. As x increases from zero, the slope of the function gets smaller. The function has an inflection point between $x=64$ and $x=96$ (the graph is too small to see exactly) where the slope of the function is zero. From that point on the function gets steeper and steeper.

The most extensive use of slopes will be in the chapters about firms (Mankiw, Ch. 13-16, but especially Ch. 13.

Review first using simple lines by viewing the Khan Academy tutorial on Slopes and Rate of Change. Play with the Math Is Fun Function Grapher and Calculator or the Math Open Reference Cubic Function Explorer. A review using calculus is in the Khan Academy tutorial on Maxima Minima Slope Intuition.