Remote Supply Revisited: The Jeep Problem with Costly Transfer Points

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Abstract

When constructing a supply chain to supply a region where resources needed for logistics activities are locally unavailable, it is not obvious how many nodes should be used in the latter ("self-sustaining") portion of the supply chain, nor how they should be positioned. This challenge arises frequently in multimodal supply chains, particularly for military operations, resource extraction, and humanitarian aid and disaster relief. It was analyzed in the mid-20th century via a classical model known as the jeep problem, and the solution involves nodes that are increasingly farther apart as they get closer to the destination. However, the solution to the jeep problem and its variants is not easily applicable to large-scale modern logistics problems. In particular, it does not work well when establishing and maintaining a node is costly and the quantity of resources to be delivered is large. In this paper, we present a modified version of the jeep problem that addresses those issues, and show that the optimal structure is equally spaced nodes over the self-sustaining portion of the supply chain. We argue that this should be used as a baseline approach for this type of supply chain. In addition, total cost is convex in the number of nodes, which ensures that finding a global optimum is tractable.

1 Introduction

The cost of logistics for operations in remote areas can be enormous. Military operations in Afghanistan, for instance, were faced with extremely high fuel costs, as described by the United States Secretary of the Navy (Mabus 2009):

"*It turns out that when you factor in the cost of transportation to a coastal facility in Pakistan – or airlifting it to Kandahar – and then you add the cost of putting it in a* *truck, guarding it, delivering it to the battlefield, and then transferring that one gallon into a piece of equipment that needs it – in extreme cases that gallon of gasoline could cost up to $400.*"

These costs are so large, in part, because the resources (fuel, in this case) must be transported via a *self-sustaining supply chain* (*SSSC*): a supply chain in which at least one resource required for logistics activities is not locally available, and is therefore provided via the supply chain itself. For example, if a truck is driving back and forth between the last two nodes, and fuel cannot be obtained at either node, the fuel consumed by the truck must itself be transported to the second-to-last node. Self-sustainment leads to a multiplier effect on resource requirements, as shown by Regnier et al. (2015): a multiplier can be associated with each stage of the supply chain, and the overall resource requirement is their product, multiplied by the end-user demand for the resource. The multipliers are determined by the lengths of the stages and the characteristics of the vehicles, including speed, capacity, and rate of resource consumption.

This phenomenon is not unique to combat or military operations. SSSCs are prevalent in humanitarian aid and disaster relief (HADR), as explored by Apte et al. (2016). The need for logistics preparation is widely recognized in HADR; see, for instance, Van Wassenhove (2006), Kovács and Spens (2007), Holguín-Veras et al. (2012), and Çelik et al. (2012). SSSCs are also common in drilling operations in undeveloped regions, and are one of many logistical challenges involved with such endeavors; see, for instance, Heimer et al. (1978), Deerhake et al. (1981), Gruenhagen et al. (2002), and Shafer (2007).

What should a supply chain for these operations look like? The classical answer to this question is provided by Fine (1947). He presents it as a *jeep problem*[1], in which a vehicle (a jeep) must cross a desert whose width is greater is than the vehicle can travel on a single tank of fuel. Solving the jeep problem is a matter of determining the optimal locations at which to establish nodes where fuel may be dropped off and stored. Fine's solution is mathematically elegant, but relies on a crucial assumption that it is costless to establish and maintain a node. While perfectly reasonable for a jeep crossing a desert, that assumption does not apply to modern global logistics challenges; fuel and other resources generally cannot be left unattended and unprotected. Facilities, personnel, and materials at transfer points are often necessary. This paper re-formulates and solves the jeep problem without the assumption of costless nodes, which leads to a qualitatively different result that is both simpler and more appropriate for large-scale real-world applications.

In most modern applications, the jeep problem does not arise in isolation. Rather, it serves as the latter portion of a multimodal supply chain, and is carried out by a different vehicle type than those used in earlier portions. There are a few settings in which this exact instance of the jeep problem arises frequently. In military operations, the jeep problem covers only the area that cannot receive external logistical support. For example, it was common for larger American bases in Afghanistan to receive supplies via airdrop, while subsequent delivery to smaller outposts was carried out by ground

[1]This problem has also been referred to as *crossing the desert* (Alway 1957, Gardner 1994), and an *exploration problem* (Ball and Coxeter 2016).

convoys (Dubbs 2011). In disaster relief settings, the jeep problem describes only the portion of the supply chain occurring in the disaster-impacted region. For example, following the 2010 Haiti earthquake, planes and ships delivered large quantities of supplies to the island, but subsequent distribution by truck was an enormous logistical challenge (Holguín-Veras et al. 2012). A lack of coordination between these distinct parts of the supply chain is frequently a major obstacle in disaster relief logistics (Kovács and Spens 2007). Maghfiroh and Hanaoka (2020) advocate for multimodal distribution for Indonesia in a disaster relief study.

However, the general problem is also an important one to multimodal transportation more broadly, as it affects planning for any routes to remote areas. In a review of multimodal freight transportation planning, SteadieSeifi et al. (2014) offer multiple reasons why SSSCs are increasingly important in this domain. Newer markets and customer bases are growing worldwide, and there is substantial environmental concern as well. The number and locations of nodes are often key considerations when new infrastructure is developed.

There are several of examples of recent studies of costs and logistical considerations in multimodal supply chains and networks where node structure is paramount. Kou et al. (2022) analyze last mile delivery in rural China, arguing for the use of different distribution modes to improve efficiency and reduce cost. Wang et al. (2020) analyze several attributes of concern to decision makers involved in multimodal transport between Wuhan and Berlin, including environmental concerns and transportation cost. One of their recommendations is to establish large container distribution centers for railway transport. Du et al. (2017) analyze road and river freight supply routes in northern Canada where water level uncertainty has a substantial impact; one of their stated goals is to provide insights about where infrastructure improvements would be the most beneficial. Kim et al. (2017) analyze the use of drones to provide the final stage of health care product delivery in rural areas, where the drone center locations are the set of decision variables in the model.

Multimodal supply chain structure is an ongoing challenge in biofuel as well. Different vehicles are used for long-haul and short-haul delivery, and the number and locations of intermediate nodes are key strategic decisions (Xie et al. 2014, Zhang et al. 2016).

An inadequate set of intermediate nodes is a common impediment to economic development of remote areas. For instance, Rahmatullah (2006) examines South Asian transportation systems, including those supplying less developed mountainous regions, and finds that multimodal transportation between countries can be extraordinarily slow; one barrier (of many) in the case of rail transportation is inadequate physical facilities at interchange points. Islam et al. (2006) explores multimodal freight transport in Bangladesh, and cites a lack of inland container-handling facilities as a major hindrance. Spandonide (2014) studies several types of transportation systems in very remote parts of Australia, and argues that the development of transport infrastructure in those areas would yield substantial economic benefits. In a study focused mostly on Ghana, Okyere et al. (2019) advocate for individual African countries to integrate multimodal transportation systems in several ways, one of which is “by developing transport links and nodes” (p. 169). For a helpful and more general review of the economic impacts of infrastructure development for several modes of transportation, see

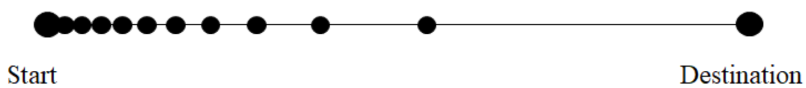

Figure 1: A notional illustration of the solution to Fine's jeep problem.

Lakshmanan (2011).

A brief summary of Fine's solution to the jeep problem is as follows. We refer to the arc between a pair of adjacent nodes as a *stage* (Phipps 1947), where the length of the stage is simply the distance between the two nodes. The first stage includes the starting point, and the last stage includes the destination:

In the solution, the lengths of the stages increase as they get closer to the destination; if n total stages are used, the length of the i^{th} stage is $\frac{1}{2(n-i)+1}$, where the distance the jeep can travel on a single tank of fuel is normalized to 1. The length of the final stage is 1; it is traversed only once. A notional illustration of this set of stages is shown in Figure 1.

Given the importance of the problem, it is somewhat surprising that Fine's work on it is not more widely known. A likely explanation is the impracticality of the solution. Several other researchers have formulated and solved extensions of the jeep problem (Phipps (1947), Franklin (1960), Gale (1970), Hausrath et al. (1995), and Wenlei et al. (2010)); all of these papers retain the costless nodes assumption and reach solutions that are analogous to Fine's.

The present paper provides a practical solution to the jeep problem that can serve as a baseline structure for remote supply. It is adapted for the sake of applicability to real-world SSSCs. It is very similar to the original jeep problem, but has the following three distinctions:

First, we require that all vehicles must be able to return once the fuel is delivered. A generalization of Fine's model can address this change easily, as discussed by Phipps (1947).

Second, Fine's jeep problem does not require any fuel to reach the destination. Modifying Fine's solution to account for this difference is straightforward, though as we will see, it can result in an impractically large number of stages.

Third, and most crucially, Fine assumes that there is no relevant cost associated with establishing and maintaining a node. Assuming costless nodes can lead to a very large optimal number of them, but no simple modification has been offered to incorporate node costs. In fact, Fine posits that with costly nodes, equally spaced nodes might be cheaper, but that including such a cost would lead to "an entirely new problem" (p. 31).

The goal of the present paper is to model and solve this new problem.

2 Model

The objective is to minimize the total cost associated with transporting fuel from a source to a destination, given a particular type of available transport vehicle. The model uses the following parameters:

- D: total distance from the source to the destination
- M: total demand for fuel at the destination
- c: total cost of establishing and maintaining a node
- g: vehicle capacity
- r: rate at which a vehicle consumes fuel (per unit distance)

To reduce notational burden, c is expressed in the same units as g and M; i.e. units of fuel rather than money. It could easily be converted to a monetary cost by multiplying by the per-unit price of fuel. It is important to note again that we are modeling the self-sustaining portion at the end of a larger supply chain. The per-unit price of fuel is partly determined by the previous mode(s) of transportation. Depending on the nature of the supply chain, it may be influenced by the market, an agreement with a contractor, organizational policy, or some combination thereof (Regnier et al. 2015). The model expresses all of the costs in terms of quantities of fuel for ease of exposition, but the actual cost of that fuel is heavily dependent on the rest of the supply chain.

We will also let N denote an upper bound on the number of stages; N can be arbitrarily large, and is included only for convenience in formulating the optimization problem.

The decision can then be framed as choosing the number and lengths of stages, or equivalently as choosing the number and locations of the nodes. We will use the former approach and let $x_1,\ldots,x_N$ denote the stage lengths. Thus, the quantity of fuel burned per vehicle round trip on stage i is $2rx_i$. Note that not all N stage lengths must be positive. If there are n positive stage lengths, we will describe this portion of the supply chain as having n stages (or, equivalently, as having $n+1$ nodes).

3 Analysis and Results

In this section, we first examine the case of costless nodes, and show that introducing a simple relaxation of the problem will dramatically change the solution. Then, we allow $c > 0$ and obtain a more intuitive result.

3.1 Costless Nodes

In the simple case where $c = 0$, the optimization problem can be expressed as:

$$\begin{aligned} \min_{x_1,\ldots,x_N} & \sum_{i=1}^{N} 2rx_ik_i \\ \text{s.t.:} & \sum_{i=1}^{N} x_i = D \\ & x_i \geq 0 \quad \forall i \\ & x_i < \frac{g}{2r} \quad \forall i, \end{aligned} \tag{1}$$

where:

$$k_i = \left\lceil \frac{M + \sum_{j=i+1}^{N} 2rx_j k_j}{g - 2rx_i} \right\rceil \tag{2}$$

is the number of round trips required on stage i. The numerator is the quantity of fuel that must be delivered to the end of stage i, and the denominator is the quantity that can be delivered on each trip. (Note also that stages of length 0 incur a cost of 0.) For $M = 0$, this problem would be equivalent to case (b) in Section 2 of Phipps (1947); a vehicle would simply have to get to the destination, with all vehicles being able to return. The problem is equivalent whether the entire process is carried out by a single vehicle or split amongst a fleet of vehicles.

The problem given by (1) can be solved using the general guidelines provided by Phipps (1947). The length x_n of the final stage will be such that k_n is minimized and x_n is maximized for that value of k_n. The minimum feasible value of k_n is $\lceil M/g \rceil$, with $M/\lceil M/g \rceil$ being delivered on each round trip, implying that:

$$x_n = \frac{g - M/\lceil M/g \rceil}{2r}. \tag{3}$$

The quantity of fuel burned on each round trip on stage n is the numerator of (3), and thus the total quantity of fuel that must be delivered to the beginning of stage n is $\lceil M/g \rceil g$. The length of each preceding stage is determined similarly. It turns out that the total quantity of fuel burned on each stage, except for the final one[2], is exactly equal to the total amount of fuel delivered to the end of that stage. The stage lengths, in backward order from stage $n-1$ to stage 1, are the terms of the harmonic series beginning with the $1+k_n$th term, multiplied by $\frac{g}{4r^2}$. As we will see in Section 3.3, this can lead to a very large number of stages.

However, a different approach to solving the problem yields additional insight. The first step is to assume that $M \gg g$; that is, the amount of fuel to be delivered is much larger than the capacity of one vehicle. Because this will necessarily lead to many round trips on each stage, we will allow non-integer values of k_i when optimizing. We do this simply by no longer applying the ceiling function. Thus, the expression for k_i will be continuous and differentiable.

This leads to a drastically different result:

Lemma 1. *If nodes are costless and non-integer numbers of round trips are permitted, then the optimal number of stages is infinite.*

The proof of Lemma 1 is in the Appendix, but the intuition is straightforward: splitting a stage into two smaller stages always reduces the total quantity of fuel burned, and there is no cost to doing so. It is specifically the requirement of integer numbers of round trips that leads to Fine's solution when nodes are costless.

[2] If the required number of stages covers a distance greater than D, then the first stage is shortened as well to account for that difference.

3.2 Costly Nodes

Given Lemma 1, we now examine the case where $c > 0$, as expressed by the following model:

$$\begin{aligned}
&\min_{x_1,\dots,x_N} \sum_{i=1}^{N} cX_i + 2rx_ik_i \\
&\text{s.t.:} \quad \sum_{i=1}^{N} x_i = D \\
&\qquad x_i \geq 0 \quad \forall i \\
&\qquad x_i < \frac{g}{2r} \quad \forall i, \\
&\text{where:} \quad k_i = \frac{M + \sum_{j=i+1}^{n} 2rx_jk_j}{g - 2rx_i} \text{for } x_i > 0 \\
&\qquad X_i = \begin{cases} 0, x_i = 0 \\ 1, x_i > 0 \end{cases}
\end{aligned} \tag{4}$$

and N is a number large enough such that a solution with more than N stages is impractical, as previously. This optimization problem is difficult to solve directly. However, we can derive a property of the solution that will both facilitate the process and serve as a useful result itself.

It will be helpful to introduce the variable γ_i: the ratio of the amount of fuel burned on stage i to the amount of fuel transported to the end of stage i. It is defined as follows:

$$\gamma_i = \frac{2rx_i}{g - 2rx_i}, \tag{5}$$

and is related to the fuel multiplier used by Regnier et al. (2015). Before finding a general solution to (4), we first state the following theorem:

Theorem 1. *Given a fixed number of stages n with non-zero length, the set of nodes that yields the minimum total fuel cost is $n-1$ equally spaced nodes, or equivalently n stages of length D/n.*

A proof is given in the Appendix. The intuition behind it is that γ_i is increasing and convex in x_i, and the relationship between the γ terms and total cost is such that when a fixed total distance is covered by two stages, cost is minimized when their lengths are equal.

Thus, for any given number of stages, we need to consider only the alternative of equal length stages. Given n stages of equal length, note that $\gamma_1 = \cdots = \gamma_n$. We will write this value as $\gamma_{(n)}$. It is decreasing in n; as the number of stages increases, the efficiency of an individual stage increases. Given Theorem 1, we are able to state that the lowest total cost achievable with n stages is:

$$cn + \left(1 + \gamma_{(n)}\right)^n M - M. \tag{6}$$

The derivation of (6) is in the Appendix, but its interpretation is straightforward. First, cn is the total node costs. The term in parentheses is the factor by which each stage is increasing the total fuel requirement (and this occurs n times). Finally, we subtract M to avoid including the end-user demand, as we are trying to minimize only the supply chain costs.

We can now simplify the full optimization problem to a obtain a problem with one decision variable (the number of stages) rather than a problem with N decision variables. The resulting optimization problem is:

$$\begin{aligned} \min_{n} \quad & cn + \left(1+\gamma_{(n)}\right)^{n} M - M \\ \text{s.t.} \quad & n > \frac{2rD}{g}, \\ \text{where} \quad & \gamma_{(n)} = \frac{2D/n}{g-2rD/n} = \frac{2rD}{ng-2rD}. \end{aligned} \tag{7}$$

The constraint on n ensures that the stages are sufficiently short for a vehicle to complete a round trip and deposit a positive quantity of fuel. With this simpler model, we are able to state the following theorem:

Theorem 2. *When stages must be of equal length, total cost is convex in the number of stages.*

The proof is given in the Appendix. Theorem 2 implies that finding the optimal value of n can be eased by treating n as real-valued; a solution falling between integers b and $b+1$ implies that the optimal arrangement is either b or $b+1$ equally spaced nodes, which can be compared directly.

3.3 Comparison of Results

Now that we have established a new method for determining an optimal number and arrangement of nodes, we can produce and compare results from this approach and from the traditional method of requiring integer numbers of round trips on each stage and assuming nodes are costless. In this section, we conduct several one-way sensitivity analyses. For each method, we illustrate the effect of D, M, and c on both the optimal number of stages and the total cost. The results are shown in Figure 2. (Sensitivity results for r and g are not shown; those parameters are characteristics of the vehicles, and their effects are qualitatively similar to those of D and M.)

The sensitivity analyses use the following set of parameter values as a baseline: $D = 1000$, $M = 4000$, $c = 10000$, $g = 100$, $r = 0.1$. The first row of charts in Figure 2 shows how the number of stages is affected in each method as D, M, and c change. When either D or M is large, the traditional approach of assuming costless nodes and requiring integer numbers of round trips leads to an enormous number of stages, while the method presented in this paper involves far fewer.

The second row of Figure 2 shows how each parameter affects total cost. A very large difference in total cost is observed when M is large. However, while the difference in total cost between the two methods is increasing in D, it is much less sensitive to D than to M. This suggests that the benefit of this paper's approach is driven more by a high demand than a long distance.

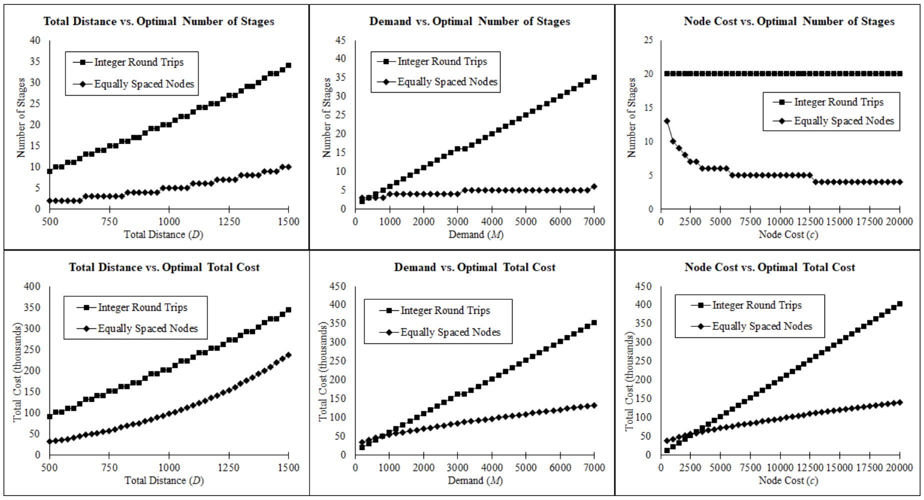

Figure 2: The effects of distance, demand, and node cost on the results of both methods

The third chart in each row shows the effect of node cost on the results. In the "integer round trips" method, costless nodes are assumed, so the structure of the supply chain is unaffected; the number of nodes is 20 regardless of node cost. However, those node costs are still incurred, which is why the relationship between c and total cost is linear. Unsurprisingly, ignoring node costs is very costly when they are large.

Recall that node cost is measured in the same units as fuel cost, which is implicitly normalized to 1 per unit. Thus, the right-hand charts also provide information about sensitivity to fuel cost, since these are the only two types of costs in the model. If there are additional logistical burdens to transport fuel to the start of the self-sustaining portion of the supply chain, that is tantamount in this model to a reduction in node cost. The less efficient the previous mode(s) of transportation, the greater the optimal number of nodes.

It is worth noting that all of the actual costs for the "equally spaced nodes" method will be slightly higher than shown in these charts, because a non-integer number of round trips is not possible in reality. However, the additional cost is very small; in the baseline scenario, the smallest number of round trips on a stage is 66.67, and rounding that up to 67 results in an additional immediate fuel cost of 13.3. (Note that the costs in Figure 2 are in thousands.) Regardless of how the nodes are arranged, it is likely in practice that vehicles will occasionally be less than completely full when leaving a node, or not deposit the precise quantity of fuel prescribed by the model, leading to small increases in the number of round trips required.

4 Discussion

This paper explores a modified version of the jeep problem in which nodes are costly, and solves the problem by relaxing the model to allow for non-integer numbers of round trips on each stage. This version of the problem is likely more applicable to modern large-scale supply in remote areas. Notably, the structure of the solution is different; cost is minimized with equally spaced nodes. In addition, cost is convex in the number of nodes, which greatly eases the search for a global optimum.

It is important to note that the model used in this paper still reflects a fairly simple logistics problem. For instance, the same type of vehicle is used throughout the entire supply chain, and the rate at which it consumes resources is constant throughout the chain as well. The solution is meant to serve as a starting point or baseline, not to be applied directly to a specific supply chain. For example, in HADR applications, it is very common for last mile distribution to be a significant challenge (Balcik et al. 2008, Beresford and Pettit 2013, John 2021), and transportation close to the destination is often much more inefficient. In such scenarios, it would likely make sense to use many short stages near the end of the supply chain.

This paper's model also considers only a route from a single source to a single destination. It is possible that expanding the problem to a transportation network with multiple sources and destinations that can share nodes would yield new results or insights. In addition to the mathematical expansion of the problem, this would increase the overall complexity of the supply chain, which would likely have further implications; see, e.g., Vachon and Klassen (2002), Choi and Krause (2006), and Craighead et al. (2007).

Finally, there is a mathematical point that merits some discussion. Consider the limit of the solution in this paper as c goes to zero. Even with infinitesimally small values of c, the optimal arrangement will still be equally spaced nodes; that is, the solution does not converge to the solution provided by Fine for $c = 0$. This disparity is due to allowing non-integer numbers of round trips. That is a trivial modeling choice when the amount of fuel to be delivered is large and many round trips are required on each stage, but it does distort results noticeably when the number of round trips on a stage is very low. Costless nodes and allowing non-integer numbers of round trips can be viewed as two possible modeling choices. The former leads to Fine's solution, and the latter leads to this paper's solution. In practice, the modeling approach that is associated with a smaller error for a given problem should be used. If establishing and maintaining a node is cheap and the quantity of resources to be delivered is small, then it is certainly possible for Fine's solution to be preferable.

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Appendix

Proof of Lemma 1. Assume the number of stages is finite, and consider an arbitrary stage i with associated $x_i > 0$ that must deliver M_i units of fuel to stage $i+1$. The amount of fuel burned on stage i is $2rx_ik_i$, or:

$$\frac{2rx_i}{g-2rx_i}M_i.$$

This means the quantity of fuel that must be delivered to the beginning of stage i is:

$$1+\frac{2rx_i}{g-2rx_i}M_i,$$

or:

$$\frac{g}{g-2rx_i}M_i.$$

This coefficient of M_i is the "multiplier" for stage i, per Regnier et al. (2015). It is given that name because these terms are multiplicative; the total fuel requirement for the chain is:

$$\left(\prod_{i=1}^{n}\frac{g}{g-2rx_i}\right)M.$$

Now, split stage i into two equal-length smaller stages by adding an intermediate node at the midpoint. To establish the Lemma, we need only show that:

$$\left(\frac{g}{g-rx_i}\right)^2<\frac{g}{g-2rx_i},$$

that is, that the quantity of fuel required at the start of stage i is lower when it is split into two equal-length stages. Since both denominators are positive per the constraints on x_i, we can multiply each side by each denominator and then divide by g, obtaining:

$$g(g-2rx_i)<(g-rx_i)^2,$$

or:

$$g^2-2grx_i<g^2-2grx_i+(rx_i)^2,$$

which simplifies to:

$$(rx_i)^2>0.$$

Since neither the rate of fuel consumption nor the stage length is zero, this condition always holds, which establishes the Lemma. □

Proof of Theorem 1. The quantity of fuel burned on stage i is γ_iM_i, where M_i is the quantity of fuel to be delivered to the end of the stage. It is straightforward to observe that $M_{i-1}=(1+\gamma_i)M_i$ for $i>1$. Applying this logic repeatedly from the end of the supply chain backward, we can state more generally that the quantity of fuel burned on stage i is:

$$\gamma_i\left(\sum_{A\subseteq\{i+1,\ldots,n\}}\prod_{z\in A}\gamma_z\right)M.$$

Summing these costs over all stages yields the following total quantity of fuel burned by the supply chain (denoted as C):

$$C = \left(\sum_{\emptyset \subset A \subseteq \{1,\ldots,n\}} \prod_{z \in A} \gamma_z \right) M.$$

Consider the equally spaced nodes arrangement, where each stage has length D/n. We will show that no deviation from this alternative can result in a lower cost. Because the sum of the lengths must be D, any increases must be accompanied by decreases of equal magnitude. Consider an arbitrary pair of stages i and j (with $i < j$), and fix the lengths of the other $n-2$ stages. We can determine the effect of changes in their lengths on total fuel requirement by taking the total derivative of C and examining only the terms involving changes in x_i and x_j. Note that $dx_j = -dx_i$, because the sum of x_i and x_j is fixed. This results in:

$$\frac{dC}{dx_i} = \frac{\partial C}{\partial x_i} + \frac{\partial C}{\partial x_j}\frac{dx_j}{dx_i} = \frac{\partial C}{\partial x_i} - \frac{\partial C}{\partial x_j}.$$

It will be convenient to compute these partial derivatives of C with respect to γ_i and γ_j:

$$\frac{dC}{dx_i} = \frac{\partial C}{\partial \gamma_i}\frac{d\gamma_i}{dx_i} - \frac{\partial C}{\partial \gamma_j}\frac{d\gamma_j}{dx_j}.$$

Consider the possible solution $x_i = x_j$. In this case, it is straightforward to show that $dC = 0$. Thus, if C is decreasing in x_i when $x_i < x_j$ and increasing in x_i when $x_i > x_j$, we will be able to conclude that for arbitrary lengths of the other $n-2$ stages, total cost is minimized when $x_i = x_j$. Writing out all of the derivatives yields:

$$\begin{aligned} \frac{dC}{dx_i} = &\left(\sum_{A \subseteq \{1,\ldots,i-1,i+1,\ldots,n\}} \prod_{z \in A} \gamma_z \right) M \frac{2rg}{(g-2rx_i)^2} \\ &- \left(\sum_{A \subseteq \{1,\ldots,j-1,j+1,\ldots,n\}} \prod_{z \in A} \gamma_z \right) M \frac{2rg}{(g-2rx_j)^2}, \end{aligned}$$

which, after simplifying and factoring common terms, yields:

$$\frac{dC}{dx_i} = 4r^2 g^2 M \left(\sum_{\emptyset \subset A \subseteq \{1,\ldots,i-1,i+1,\ldots,j-1,j+1,\ldots,n\}} \prod_{z \in A} \gamma_z \right) \left[\frac{x_i - x_j}{(g-2rx_i)^2 (g-2rx_j)^2} \right].$$

The term outside the square brackets is positive, and the denominator of the term inside the square brackets is positive as well. Thus, the sign of the derivative is determined by $x_i - x_j$; the total quantity of fuel burned is increasing when $x_i > x_j$ and decreasing when $x_i < x_j$. Therefore, we can conclude that the minimum total fuel cost is achieved when $x_i = x_j$.

Since i and j were chosen arbitrarily, it must be true that if any two stages must cover some (feasible) fixed total distance, it is done with minimum fuel cost by choosing equal lengths for the two stages. Thus, any $x_1, \ldots, x_n$ such that $x_1 = \cdots = x_n$ does not hold must be suboptimal, because the total fuel cost can be reduced by equalizing the lengths of two unequal stages. Thus, it must be true that the optimal lengths satisfy $x_1 = \cdots = x_n = D/n$, establishing the Theorem. □

Derivation of (6). As shown in the proof of Theorem 1, the total quantity of fuel burned by the supply chain is:

$$\left(\sum_{\emptyset \subset A \subseteq \{1,\ldots,n\}} \prod_{z \in A} \gamma_z \right) M.$$

However, since all stages are now of equal length, every instance of γ_z can be replaced by $\gamma_{(n)}$. Adding the node cost cn and simplifying the total fuel cost yields:

$$C = cn + \left(\sum_{\emptyset \subset A \subseteq \{1,\ldots,n\}} \gamma_{(n)}^{|A|} \right) M.$$

(For notational convenience, we modified the interpretation of C slightly here to reflect the total supply chain cost, i.e. including node costs, as opposed to the fuel costs only.) Alternatively, this can be written as:

$$C = cn + \left(\sum_{i=1}^{n} \binom{n}{i} \gamma_{(n)}^{i} \right) M.$$

This summation is the binomial expansion of $\left(1 + \gamma_{(n)}\right)^n$, with the constant term of 1 removed. Thus, total cost can be rewritten as:

$$cn + \left(\left(1 + \gamma_{(n)}\right)^n - 1 \right) M,$$

or:

$$cn + \left(1 + \gamma_{(n)}\right)^n M - M.$$

□

Proof of Theorem 2. The total cost for n equal-length stages is given by:

$$cn + \left(1 + \gamma_{(n)}\right)^n M - M.$$

Taking the derivative with respect to n yields:

$$c + \left(1 + \gamma_{(n)}\right)^n \left(\ln(1 + \gamma_{(n)}) + \frac{1}{1 + \gamma_{(n)}} n \gamma_{(n)}' \right) M, \tag{8}$$

It will be helpful to compute $\gamma_{(n)}'$, which is:

$$\gamma_{(n)}' = \frac{-2rgD}{(ng - 2rD)^2}$$

After expanding the rightmost $\gamma_{(n)}$ term in (8), this allows us to simplify the derivative of total cost to:

$$c + \left(1 + \gamma_{(n)}\right)^n \left(\ln(1 + \gamma_{(n)}) - \gamma_{(n)} \right) M,$$

The second derivative is then:

$$\left(1+\gamma_{(n)}\right)^n \left[\frac{\gamma'_{(n)}}{1+\gamma_{(n)}} - \gamma'_{(n)} + \left(\ln(1+\gamma_{(n)}) - \gamma_{(n)}\right)^2\right] M,$$

Since $\left(1+\gamma_{(n)}\right)^n$ and M are both positive, we need only show that the expression in the square brackets is positive. By expanding $\gamma_{(n)}$ and $\gamma'_{(n)}$, the first two terms can be combined and simplified, yielding:

$$\left[\frac{-\gamma_{(n)}\gamma'_{(n)}}{1+\gamma_{(n)}} + \left(\ln(1+\gamma_{(n)}) - \gamma_{(n)}\right)^2\right].$$

Both of these terms are positive as well (note that $\gamma'_{(n)}$ is negative), which means the second derivative is positive, and thus total cost is convex in the number of stages, establishing the Theorem.

$\square$