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The Legacy of Peter Fishburn: Foundational Work and Lasting Impact

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Abstract. Peter Fishburn has had a tremendous impact on the field of decision analysis, developing ideas that would come to be foundational across decision analysis and that would impact the literature on decision making in economics, psychology, finance, engineering, and mathematics. This paper provides an overview of his legacy. We summarize 11 of his influential papers. We then trace his impact on recent research in topics including preference representation and elicitation, risk attitudes, time preferences, health preferences, behavioral decision making, social choice and voting, and geometric analyses.

Supplemental Material: The online appendix is available at https://doi.org/10.1287/deca.2022.0461.

Keywords: utility theory • normative models • behavioral decision making • group decision making • preference modeling

1. Introduction

Whenever I would start in a new field, I would look at the literature, and I would find foundational papers by Peter.
—Robin Keller

The field of decision analysis has a rich history, with a strong foundation built from its modern roots in the work of Ramsey (1931), von Neumann and Morgenstern (1947), and Savage (1954). Among the most prolific and influential contributors to the foundation of the field was Peter Clingerman Fishburn (1936–2021), whose breadth of work is a testament to the breadth and impact of decision analysis. His research spanned preference theory, preference elicitation, social choice, and group decision making, among many other areas. For much of his career, he was employed at Bell Laboratories. In honor of his contributions to the field, he received the Frank P. Ramsey Medal, the highest award of the Decision Analysis Society (DAS), in 1987, and the DAS Publication Award for *Choice Under Uncertainty* (Fishburn and LaValle 1989), edited by Fishburn and Irving H. LaValle, in 1991. However, his influence was by no means limited to the decision analysis community. He received an honorable mention for the Frederick W. Lanchester Prize for *Utility Theory for Decision Making* (Fishburn 1970a), and was the winner of the John von Neumann Theory Prize in 1996 for his overall body of work.

Fishburn was honored with a memorial panel at the 2021 INFORMS Annual Meeting, in which several prominent decision analysis scholars recounted his legacy and direct influence on their work. The quotations at the beginning of each section of this paper are from speakers on that panel (included with permission). His biographical profile appears on INFORMS' website in recognition of "significant contributions to operations research and the management sciences" (INFORMS 2021). His legacy is honored in an obituary written by three of his coauthors and published in *Social Choice and Welfare* (Brams et al. 2021), and will be commemorated in an upcoming special issue of the *Journal of Mathematical Psychology* (Regenwetter et al. 2022).

We did not know Fishburn personally, and our academic careers did not overlap with his. However, we encounter and benefit from his contributions frequently; it is nearly impossible for a scholar or practitioner in decision analysis and related fields to avoid being influenced by him in some way, whether directly via a concept from

one of his publications or indirectly through subsequent research that builds on his work. Our goal in writing this paper is to organize, catalog, and summarize Fishburn's contributions to help ensure that his legacy continues to be recognized and appreciated by future generations.

We close this introduction with a disclaimer: it is not possible to capture the full impact of Fishburn's work in this paper. According to citation data[1] collected using Publish or Perish software (Harzing 2007) to collect citation counts from Google Scholar, his papers and books have been cited over 43,000 times as of August 2022[2] and collectively average over 1,000 citations per year. He has 80 separate works that have been cited at least 100 times and has authored or coauthored over 500 works. Section 2 provides summaries of 11 of Fishburn's most influential papers. Section 3 explores the breadth of his work, discussing its impact on several different fields and compilations in his many books. Finally, Section 4 concludes this paper.

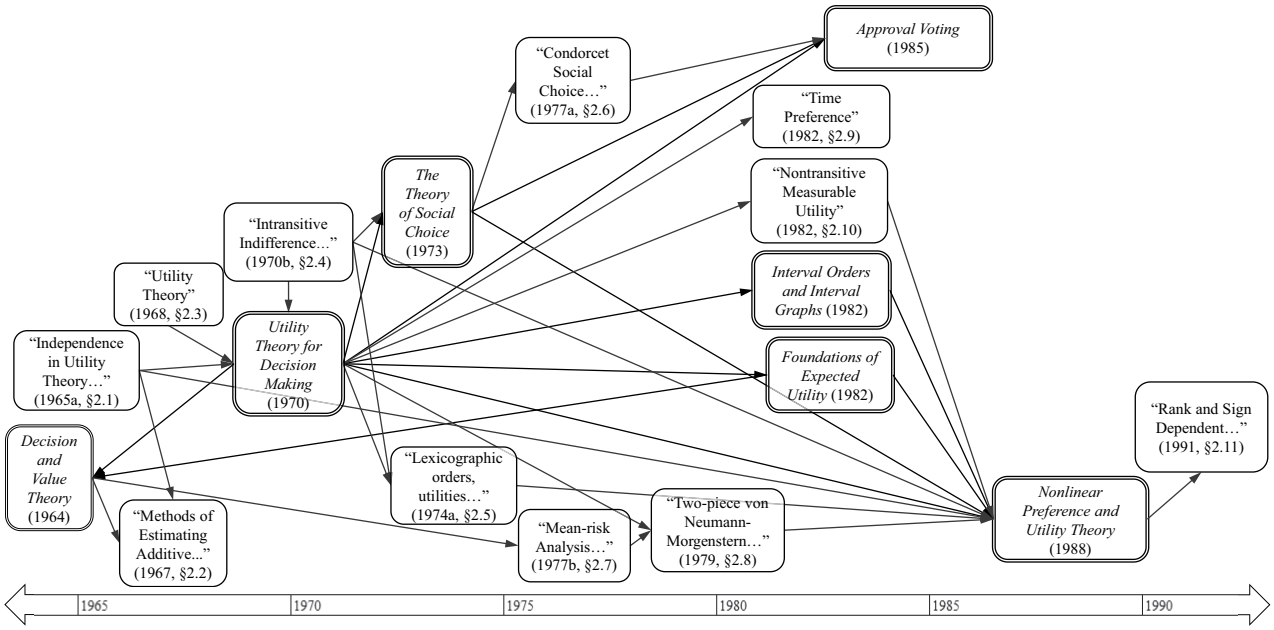
2. Summaries of Selected Papers

He was always someone who did more than his share.
—Ralph Keeney

In this section, we provide brief summaries of 11 of Fishburn's especially influential papers. For each paper, our goal is to convey high-level explanations of both the paper's topic and the reason(s) why it is frequently cited. Also included with each summary is a sample of subsequent prominent works (by other authors) that cite the paper.

Fishburn published several influential books as well, which are not included. Much of the material in the books overlaps with material in his individual papers; a mapping between the two is provided in Figure 1. Although we restrict our summaries to Fishburn's work as presented in original research articles, we note one contribution of the superb and widely praised book *Utility Theory for Decision Making* (Fishburn 1970a) here. It gave the first complete proof of Savage's (1954) theorem, a critical part of the foundation of decision analysis. In particular, Fishburn exactly specified the domain of preference and proved that utility must be bounded. In addition, Fishburn authored several literature reviews and reflections (Fishburn 1981; 1988; 1989a, b; 1991). Two of his most influential reviews reflect on the accomplishments and future challenges in multiple criteria decision making and multiattribute utility (Dyer et al. 1992, Wallenius et al. 2008).

Figure 1. A Mapping of Citations Between Fishburn's Books (Double Lined) and the Papers (Single Lined) Summarized in This Section



Note. Arrows point toward the subsequent, citing work.

When selecting the set of influential papers to summarize, we considered five factors: (1) overall impact as measured by citation count, (2) direct impact on recent decision analysis work as shown by citations in *Decision Analysis* (2004–present), (3) direct impact on recent work in related fields as shown by citations in related journals (2004–present),[3] (4) Fishburn's implicit assessment as evidenced by citations within Fishburn's own books, and (5) overall coverage in terms of how distinct a paper's contribution is from other papers in the set. In most cases, these factors were not in conflict with one another, but some judgment calls were necessary. Inasmuch as disagreement exists on this set of papers, it can be viewed as a testament to the scope and impact of his work.

2.1. "Independence in Utility Theory with Whole Product Sets" (Fishburn 1965a; 448 Citations)

This is a seminal multiattribute preference theory paper. It serves as a foundation for subsequent theory and applications involving additive multiattribute utility functions.

This paper provides independence conditions that allow multiattribute preferences under uncertainty to be represented by an additive utility function. That is, interactions between attributes such as substitution or complementarity are precluded. The conditions are straightforward and include only 50–50 gambles; no other probabilities are needed. The intuitive requirement is that preferences over gambles must depend only on the marginal distributions for each attribute, and not the joint distribution. Several easily accessible examples are provided.

This work gave birth to multiattribute utility theory, which is central to decision analysis. It supports prominent early work by decision analysts (Raiffa 1969, Keeney and Raiffa 1976), and is foundational to additive representations of preferences (Wakker 1989). It also supports models of preference for more specific settings, such as trade-offs between health and life years (Pliskin et al. 1980).

2.2. "Methods of Estimating Additive Utilities" (Fishburn 1967; 594 Citations)

This paper is invaluable when trying to determine how multiattribute preferences under uncertainty should be assessed in a given decision setting. While most of Fishburn's works are original contributions, this paper is a helpful survey of 24 different elicitation methods for additive multiattribute value and utility functions. Among many other works, it cites six of Fishburn's own prior publications that include original contributions (Fishburn 1964; 1965a, b; 1966a, b, c). The paper provides simple and concise explanations of each method, and classifies them in several important ways. A rich table in the paper states, for instance, whether each method requires the use of probabilities.

The survey has been widely cited, including by work on measurable value functions (Dyer and Sarin 1979), incorporating regret in decision making (Bell 1982), rough sets (Greco et al. 2001), and overviews of decision analysis (Keeney 1982).

2.3. "Utility Theory" (Fishburn 1968; 628 Citations)

This paper provides a comprehensive description of utility theory and the literature up to that point, with an emphasis on "a prescriptive interpretation of utility theory" (Fishburn 1968, p. 336). The literature review describes interpretations of utility theory and provides details on theoretic results pertaining to preference orders and utility functions, alternatives with multiple attributes, time preferences, utility differences, expected utility, subjective probability, and social choice.

The paper has been widely cited as a reference for the foundations of utility theory. It is cited by research in decision analysis that describes utility functions and their assessments (Keeney 1970, Farquhar 1984, Harvey 1988). It has also garnered citations across a wide range of related fields and applications, including engineering (Hazelrigg 1998, Georgiadis et al. 2013), machine learning and artificial intelligence (Pavlov and Marinov 2018, Vamplew et al. 2018), information systems (Jutila and Baram 1971, Shkarlet et al. 2019), optimization (Song and Roy-Chowdhury 2008, Krokhmal et al. 2013), economics (Zavadskas and Turskis 2011), pricing (Smith and Nau 1995, Berrada et al. 2018), and risk analysis (Boakye et al. 2022), with the breadth of citing fields reflecting the breadth of application of utility theory and decision analysis.

2.4. "Intransitive Indifference with Unequal Indifference Intervals" (Fishburn 1970b; 531 Citations)

This short paper provides a representation theorem for an interval ordered preference relation. The paper begins by presenting interval orders as more restrictive than strict partial orders, but less restrictive than semiorders. It explains the concept of indifference intervals as a property

of a utility function: for any x, there is an indifference interval surrounding $u(x)$, and x and y are indifferent if and only if their intervals overlap. A semiorder can be represented by a utility function with equal-sized indifference intervals. The main theorem of the paper states that interval orders can be represented by utility functions with indifference intervals whose sizes are a function of x.

Interval ordered preference relations concern violations of transitivity to various degrees (e.g., Jamison and Lau 1973, Doignon 1987) and can be useful when a decision maker cannot discriminate perfectly between similarly preferable alternatives (Luce 1996, Aleskerov et al. 2007). The paper is also cited in work on graph theory (e.g., McKee and McMorris 1999) because of its connection to interval graphs (Suppes et al. 1989), and it is useful for measures of physical sensation (Falmagne 2002).

2.5. "Lexicographic Orders, Utilities and Decision Rules: A Survey" (Fishburn 1974a; 894 Citations)

This paper provides a thorough survey of lexicographic orders and preferences, as well as two axiomatizations. The paper first reviews some basic underlying concepts and terminology. It then provides two different sets of axioms for lexicographic preferences, pointing out the restrictiveness of such preferences. Several less restrictive implementations are discussed, including partial orders, satisficing, and constrained optimization. The challenges of representing lexicographic preferences with a real-valued function are also explored, as well as the use of additive representations for probability measures, vector spaces, and expected utility.

This paper is frequently cited as a principal reference for lexicographic ordering. Prominent citing work includes Svenson's (1979) system representing the process of decision making, examinations of decision heuristics (Gigerenzer and Todd 1999, Shah and Oppenheimer 2008), reviews of the decision-making literature (Schoemaker 1982), and work studying consumer preferences (Kohli and Jedidi 2007).

2.6. "Condorcet Social Choice Functions" (Fishburn 1977a; 711 Citations)

The Condorcet principle states that the candidate or option that beats every other candidate under simple majority should be selected, provided one exists. In this work, Fishburn considers this principle and examines nine social choice functions proposed to handle situations in which there is no majority candidate. The majority of functions meet desirable properties such as homogeneity, monotonicity, and Pareto optimality. More stringent properties are examined, including Smith's (1973) variation of the Condorcet principle, and the inclusive and exclusive Condorcet principles, which are designed for situations with no majority candidate but several ties. In considering the trade-offs in performance, Fishburn posits that if monotonicity, Pareto optimality, and Smith's (1973) variation of the Condorcet principle are required, then only Copeland's (1951), Kemeny's (1959), and Fishburn's (1977a) functions remain in contention. Of these, Fishburn's (1977a) function satisfies the most additional conditions but scores poorly on discriminability. Kemeny's (1959) does best among these three if discriminability is important, but in some cases this function can be difficult to compute, in which case Copeland's (1951) function may be preferred.

Prominent citing work includes an extension of Condorcet's principle (Young and Levenglick 1978), as well as work on voting systems (Brams and Fishburn 1978, Bartholdi et al. 1989, Nurmi 1999), group decision making (McKelvey 1986, Sen 1986, Hou and Triantaphyllou 2019), and tournaments (Moulin 1986, Berghammer et al. 2013, Brandt et al. 2016).

2.7. "Mean-Risk Analysis with Risk Associated with Below-Target Returns" (Fishburn 1977b; 1,947 Citations)

This paper examines a general model to represent preferences in target-based decisions over investment returns. A special case of the general model is the two-parameter α–t model, where α can be varied to represent a variety of risk attitudes. Fishburn derives three theorems related to the characteristics of the model, describing (1) properties of the risk attitude for conditions related to the mean return and probability below the target, (2) conditions for congruence with expected utility maximization and approximations of parameters for empirically assessed utility functions, and (3) how the model relates to first- and second-order stochastic dominance.

The paper has been influential on subsequent research involving risk attitudes and risk taking (Laughhunn et al. 1980, March and Shapira 1987, MacCrimmon et al. 1988, Mitchell 1999), particularly in the financial risk literature.

Many subsequent risk measures are either special cases of Fishburn's measure or are closely related (Dowd 2005). The development of the widely used mean-risk (Ogryczak and Ruszczyński 1999, Gotoh and Konno 2000, Roman et al. 2007) and value-at-risk models (Benati and Rizzi 2007, Kuan et al. 2009) follows.

The impact of this paper extends beyond finance. It has been influential in the development of target-adjusted utility models (Schneider and Day 2016) and the use of lower partial moments to measure risk perception (Unser 2000). Delquié (2012) proposes a risk measure based on the expected value of perfect information and shows that it relates to Fishburn's (1977b) model.

Among all citing work, the most prominent is Kahneman and Tversky's (1979) prospect theory, which cites Fishburn (1977b) as a predecessor, which is important because it already involves sign dependence. As Fiegenbaum and Thomas (1988) and Fiegenbaum (1990) emphasize, targets can be conceptualized as reference points. This interpretation of targets further connects the two works.

2.8. "Two-Piece von Neumann-Morgenstern Utility Functions" (Fishburn and Kochenberger 1979; 607 Citations)

This paper examines how well simple functional forms, including linear, power, and exponential functions, can represent empirically assessed utility functions, where separate functional fits are made below and above a target level. The paper examines 28 empirically assessed utility functions and finds the most common pattern of curvature (13 of 28) is convexity below target and concavity above target. Power and exponential functions provided the best fits to the data, with power functions tending to represent the more flatly sloped data better, and exponential functions representing the more steeply sloped data better. Finally, the results show generally steeper utility curves below target than above.

Fishburn and Kochenberger (1979) explain that these findings lend support to Kahneman and Tversky's (1979) work, which was still forthcoming at the time, and which in turn cites this work. Subsequent work related to prospect theory and descriptive decision analysis often cites this paper (Tversky and Kahneman 1985, 1989; Rabin 1998; Trepel et al. 2005). Thus, Fishburn's work contributed to loss aversion, one of the main components of prospect theory.

This paper continues to influence the literature as researchers examine representations of preferences under cumulative prospect theory (Pennings and Smidts 2003, Stott 2006, Booij and van de Kuilen 2009, Harrison and Swarthout 2016). The development of generalized models of disappointment (Jia et al. 2001) builds on this work, as does research examining risk attitudes (Kirkwood 2004, Baucells and Villasís 2010). Reference points also arise in a range of applications, including negotiation (Larrick et al. 2009), stochastic network equilibria (Avineri 2006), and financial asset performance (Pedersen and Rudholm-Alfvin 2003).

2.9. "Time Preference" (Fishburn and Rubinstein 1982; 562 Citations)

This paper provides an axiomatic foundation for time discounting, important in numerous applications in economics and many other fields. It begins with an accessible formulation of time-discounted preferences. Four simple axioms permit a continuous value function that is monotonic in outcomes, decreasing in time for desirable outcomes, and increasing in time for undesirable outcomes. A stationarity axiom allows for a value function that applies exponential discounting to a single-attribute value function over the outcomes, while, alternatively, a weaker Thomsen condition allows for separability of outcomes and time. A similar result is presented using gambles over outcome/time pairs.

This paper is widely cited as a foundation for time preference and discounting (Barro and Sala-i-Martin 1995), with citations spanning issues and topics such as observed anomalies (Loewenstein and Prelec 1992, Frederick et al. 2002, Benhabib et al. 2010, Halevy 2015), models with nonconstant discount rates (Rohde 2010, Montiel Olea and Strzalecki 2014), and discounting of probabilities (Baucells and Heukamp 2012).

2.10. "Nontransitive Measurable Utility" (Fishburn 1982a; 601 Citations)

In the early 1980s, there was growing interest in generalizations of expected utility. Sections 2.7 and 2.8 showed Fishburn's influence on prospect theory, which was one such generalization. Another important direction concerned the relaxation of transitivity. This paper was one of the important initiators, simultaneously and independently of Loomes and Sugden (1982).

The paper begins by presenting continuity, dominance, and symmetry axioms for preferences under uncertainty. The main theorem of the paper states that these axioms allow for a measurable function that represents preferences; specifically, it is a *skew-symmetric bilinear* function on pairs of gambles. Roughly speaking, it measures the intensity of preference for the first gamble over the second gamble. The proof of the theorem comprises the majority of the paper.

The paper is cited frequently by subsequent work on regret theory (e.g., Loomes and Sugden 1987, Sugden 1993, Quiggin 1994). It is also cited in empirical tests of transitivity (Baillon et al. 2015, Roelofsma and Read 2000), as well as a prominent review of nonexpected utility theories (Starmer 2000).

2.11. "Rank- and Sign-Dependent Linear Utility Models for Finite First-Order Gambles" (Luce and Fishburn 1991; 408 Citations)

In this paper, Luce and Fishburn axiomatize first-order gambles in a way that integrates prospect theory (Kahneman and Tversky 1979) and rank-dependent theories (Quiggin 1982; Luce 1988, 1991). In modern prospect theory, preferences depend on both the rank order of consequences and the relation of a consequence to the status quo (Tversky and Kahneman 1992). The axiomatization in this paper includes a plausible descriptive axiom that decomposes a gamble to the joint receipt of its gains relative to the status quo and its losses relative to the status quo. This paper extends these results to first-order gambles with *any* finite number of outcomes and gambles of uncertain events in which the probabilities are not known. The result of this work is extremely similar to *cumulative prospect theory*, as presented by Tversky and Kahneman (1992), and is perhaps not as widely recognized as it deserves to be.

The axiomatization of a joint receipt operation distinguishes the work in this paper. The joint operation is denoted by $\oplus$, where $a \oplus b$ indicates that both consequences a and b are received. This operation provides a structure of ordered concatenation that can treat gains and losses separately, enabling a formal means to represent the editing of gambles. This editing, of course, is not normative, which causes one of the axioms to be described as "plausible."

The contributions of the paper are important to the literature on descriptive decision analysis (Busemeyer and Townsend 1993, Starmer 2000), with particular impact on work dealing with probability weighting functions (Camerer and Weber 1992, Prelec 1998, Gonzalez and Wu 1999, Kilka and Weber 2001, Cavagnaro et al. 2013).

3. Impact on Decision Analysis and Related Fields

> *Before you get too carried away … you might be surprised to find out that Peter thought about those ideas a long time ago, and it would be worthwhile to see what he had to say.*
>
> —Jim Dyer

The magnitude and breadth of Fishburn's impact are immense. He has 20 works with over 500 citations each. (For details, see the cleaned citation data set that accompanies this paper in the online appendix.) The goal of this section is to convey his lasting direct impact on areas of research in both decision analysis and related fields such as management science, economics, and psychology. Only papers that cite Fishburn are included. Particular attention is given to more recent citations, because they provide unambiguous evidence of the continuing impact of his work. The remainder of this section is divided into seven subsections, each presenting a brief summary of Fishburn's enduring impact on a particular subfield.

3.1. Representations of Preferences

Fishburn published several foundational papers establishing that preferences obeying given sets of conditions can be represented by utility functions with resulting properties, for example, Fishburn (1965a) and Fishburn (1982a), which are summarized in Section 2. There is a wide range of recent research on preference theory that builds directly upon his work.

Many advances in multiattribute utility theory fall into this category, including the development of new independence concepts (Abbas and Howard 2005, He et al. 2014, Leonelli and Smith 2017), reference-dependent models (Abdellaoui et al. 2007, Bleichrodt et al. 2009), ambiguity (Borgonovo and Marinacci 2015, Baillon et al. 2017), performance targets (Bordley and LiCalzi 2000, Bordley and Kirkwood 2004), and habit formation and satiation (He et al. 2013, Baucells and Zhao 2020).

Fishburn's contributions to modeling preferences also support expansions of value and utility functions into domains with more complex sets of outcomes. Examples include portfolio decisions (Morton 2015, Liesiö and Vilkkumaa 2021), spatial preferences (Simon et al. 2014, Harju

et al. 2019, Malczewski and Jankowski 2020), and altruistic preferences (Bell and Keeney 2009, Simon 2016).

There is ample research building on Fishburn's work on indifference intervals; see, for example, Bouyssou and Pirlot (2004), Candeal and Induráin (2010), and Candeal et al. (2012). Finally, Borie (2016) expands on Fishburn's work on lexicographic preferences.

3.2. Preference Elicitation

The body of recent work on preference elicitation has multiple connections to Fishburn. First, preferences may be elicited to fit a value or utility function whose existence is supported in part by his work. There are many such papers; some examples in this journal include elicitation for general multilinear utility functions (Montiel and Bickel 2014), preferences that involve equity (Taheri and Wang 2018), and several different types of utility functions that owe theoretical foundations to Fishburn (Wakker et al. 2004).

Recent work on specific applications of additive multiattribute preferences often refers to Fishburn (1967), summarized in Section 2, when discussing elicitation methods. Examples include decisions related to hospital wastewater treatment (Schuwirth et al. 2012), recommender systems (Scholz et al. 2017), ecosystem services (Mavrommati et al. 2017, Borsuk et al. 2019), cruise ship routes (Pesce et al. 2018), and urban regeneration (Ferretti and Grosso 2019).

Finally, and most directly, Fishburn is also cited by papers whose main contributions are new approaches to preference elicitation. For instance, de Almeida et al. (2016) develops an interactive method for weight assessment. Anderson and Clemen (2013) introduces a three-step process to mitigate some of the practical shortcomings that arise when eliciting multiattribute preferences. Additionally, there are examples of automated elicitation approaches in the computation literature (Braziunas and Boutilier 2005, Branke et al. 2017).

3.3. Analyses of Risk Attitudes

The varying ways that decision makers respond to risk provides an active area of research in decision analysis. Section 2.7 summarizes Fishburn's (1977b) study of risk associated with targets. The research led to many applications incorporating risk attitudes, including the ranking and selection of financial assets, such as the work of Farinelli and Tibiletti (2008, p. 1543), who explain that "the seminal idea of expressing different attitudes by means of different orders can be traced back to Fishburn (1977b)." Other applications in finance include enhanced indexation (Lejeune 2012), robust portfolio selection (Chen et al. 2011), and portfolio optimization to meet a target (Roman et al. 2006, Comova and Nawrocki 2014, Zakamouline 2014, Lwin et al. 2017, Warren 2019). Additional applications building on the work include modeling agents' risk attitudes in a supply chain (Gan et al. 2005, Basu et al. 2019, Choi et al. 2018), managing supply chain risk (Hahn and Kuhn 2012), and examining the newsvendor problem with different approaches to risk (Gotoh and Takano 2007, Rubio-Herrero and Baykal-Gürsoy 2020).

Fishburn's body of work also examines other risk measures that consider the probability distribution over loss (Fishburn 1984) and the effect of gains (Fishburn 1982b). This work is cited in the proposal of a standard measure of risk that depends on an individual's utility function and, as a result, is consistent with expected utility (Jia and Dyer 1996). Analyses of decisions based on risk–value trade-offs follow (Butler et al. 2005, Jia and Dyer 2009).

In another line of work on risk attitudes, Bell and Fishburn (2001) present a strong one-switch condition to describe risk preferences that can switch at most once as wealth increases. Literature building on this work includes work on one-switch risk preferences in multivariate settings (Abbas and Bell 2011), higher order risk effects (Deck and Schlesinger 2010), sensitivity to risk aversion (Sandvik and Thorlund-Petersen 2010), and issues of preference reversals (Bakır and Klutke 2011).

Descriptive work on modeling risk attitudes has also been influenced by Fishburn and Kochenberger's (1979) work on modeling risk attitudes above and below a reference point, which greatly influenced prospect theory (Kahneman and Tversky 1979, Tversky and Kahneman 1992). The piecewise representation, along with the concept of reference points, influences the development of descriptive loss aversion indexes (Köbberling and Wakker 2005, Charles-Cadogan 2016) and the representation of risk aversion in applied settings such as entrepreneurial decisions (Hamböck et al. 2017).

3.4. Health and Life Decisions

Much like the influence on modeling preferences generally, Fishburn's (1965a) work on utility independence has been cited as foundational to health utility measurement (Bleichrodt and Pinto 2012) and, in conjunction with Fishburn and Keeney (1975), has supported the

development of models representing health and life decisions. In particular, quality-adjusted life years (QALYs), one of the most central outcome measures used in health, rely heavily on these contributions (Pliskin et al. 1980). Advances include multiattribute techniques for obtaining QALYs (Feeny et al. 1995, Torrance et al. 1995), considering extrinsic goals in analysis involving QALYs (Hazen 2007, Hazen and Schwartz 2009), analysis of major life decisions (Keeney and Vernik 2007), and the development of multiplicative models over health and wealth states (Lichtendahl and Bodily 2012).

There has also been some recent analysis of the preference assumptions and elicitation methods associated with QALYs. For instance, Spencer and Robinson (2007) find evidence supporting utility independence in health decisions. It may also be possible to improve upon the time trade-off method commonly used to assess preferences in this setting (Janssen et al. 2013, 2021).

3.5. Behavioral Decision Making

Fishburn contributed substantially to the early literature on intransitive preferences, attribute reference levels, and lexicographic ordering. His work is often referenced in behavioral papers that study these and related behavioral patterns in decision makers.

His contributions support a large body of recent work that compares the empirical validity of multiple nonexpected utility methods (Day and Loomes 2010, Kogler et al. 2013, Baillon et al. 2015, Birnbaum and Diecidue 2015, Birnbaum et al. 2016, Pachur et al. 2017, Herweg and Müller 2021). In general, these papers are descriptive in nature, but some still have prescriptive goals (Katsikopoulos 2011, Keller and Katsikopoulos 2016, Bhatia 2018).

Fishburn's work is also cited in recent papers examining the impact of experience or repetition on violations of expected utility (Birnbaum and Schmidt 2010, Birnbaum and Schmidt 2015, Harman and Gonzalez 2015).

In addition, Fishburn is frequently cited in recent papers on consumer behavior, including some examining the impact of regret (Diecidue et al. 2012, Chorus et al. 2014, Lyons et al. 2019), as well as the implementation of lexicographic preferences (Zhu and Timmermans 2011, Evangelidis and Levav 2013, Jung et al. 2015).

3.6. Social Choice and Group Decision Making

Many group-decision-making methods have been proposed in the literature. In the absence of a clear majority, the difficulty in determining the best method is well illustrated by Fishburn's (1977a) analysis of the Condorcet principle: "although some methods are better than others, there is no obviously best method" (Brams and Fishburn 1978, p. 831). To address this difficulty, and following up on Section 2.6, Brams and Fishburn (1978, 1983) examine approval voting, a nonranking mechanism in which voters can vote for as many candidates as they wish, but cannot cast more than one vote for any single candidate.

The work on approval voting has been foundational in the literature on social choice broadly (e.g., Sen 1986, 1999; Elster and Hylland 1989; Arrow 2012) and has fostered much additional work in approval voting (Weber 1995, Laslier and Sanver 2010). The body of research has led to the implementation of approval voting in some municipalities (Brams and Fishburn 2005, Laslier and Van der Straeten 2008) and in organizations, including voting for society officers in INFORMS and in the Society for Judgment and Decision Making (SJDM).[4] The work continues to influence research on voting, including work examining challenges and strategic gaming of approval voting (Niemi 1984, Endriss 2013, Bassi 2015, Meginnis et al. 2021), comparisons of approval voting and pluralities (Bouton et al. 2016, Igersheim et al. 2022), and committee selections with approval-based rules (Aziz et al. 2017, Elkind et al. 2017), and provides a basis for analyses of social choice systems such as participatory budgeting (Benade et al. 2021).

Approval voting has influenced the development of new social choice mechanisms such as preference approval voting, which combines information from approval and rankings (Brams and Sanver 2009), and work examining the properties of preference approval voting (Dong et al. 2021). Beyond approval voting, Fishburn's (1965a) axiomatic work on mutual utility independence has also been used as a foundation for further work in group decision making (Keeney 2013).

3.7. Geometric Analyses of Decision Making

Although less common in decision theory, geometry can sometimes be useful in solving decision problems. Fishburn's work in this area has provided useful results for many areas of decision research. For example, his work with Fiorini (Fiorini and Fishburn 2004) on weak order polytopes contributes to a formulation for preference aggregation in group decision making (Yoo and Escobedo 2021, Escobedo et al. 2022). It also relates to

work using hyperplanes in preference modeling (Ovchinnikov 2005) and work on mixture models of order relations (Doignon and Rexhep 2016). His work with LaValle (Fishburn and LaValle 1995) using a finite, unidimensional grid to analyze special stochastic dominance relations relates to subsequent work involving orderings, including the Laplace transform order (Denuit 2001) and stochastic dominance relations (Chakravarty and Zoli 2012).

Fishburn's (1973a, 1974b) development of a diagonal decomposition of utility functions is generalized by Farquhar (1975) to enable the decomposition of utility functions. Together, these contributions support further developments in representing multiattribute preferences with utility functions (Abbas and Howard 2005, Abbas 2010, Liesiö and Vilkkumaa 2021, Xie and Zhou 2022) and in representing preferences with spatial components (Keller and Simon 2019).

4. Conclusion

If you keep on reading his papers … you learn from it, you benefit from it.

—Peter Wakker

New generations of decision analysis researchers will not have the opportunity to work directly with Fishburn, but there is a tremendous benefit to reading his work. He has gifted researchers with a bounty of papers and books from which to learn and to be inspired. Whatever the in subfield of decision analysis or a related field in which a person is involved, the chances are high that Fishburn has studied pertinent ideas and has provided a solid foundation on which to build new avenues of inquiry. We hope that this paper motivates researchers to explore his work further.

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Appendix

The selection of articles considered the impact, as measured by number of citations since 2004, within a set of 19 prominent journals that often publish papers related to decision analysis: *Decision Analysis*, *Decision Support Systems*, *Judgment and Decision Making*, the *Journal of Behavioral Decision Making*, *Multiple Criteria Decision Analysis*, *Organizational Behavior and Human Decision Processes*, *Theory and Decision*, *Medical Decision Making*, the *American Economic Review*, *Econometrica*, the *Journal of Economic Theory*, the *European Journal of Operational Research*, the *Journal of Risk and Uncertainty*, *Management Science*, *Operations Research*, *Risk Analysis*, the *American Journal of Political Science*, the *American Political Science Review*, and the *Journal of Mathematical Psychology*. The fifteen most cited journal articles by Fishburn in these journals are listed in Table A1.

Table A1. Fifteen Most Frequently Cited Articles by Fishburn in the Set of Related Journals, with Rank and Total citation Count (in Parentheses) as of March 2022.

Article	Rank
Fishburn (1977b)	1(131)
Wallenius et al. (2008)	2(127)
Fishburn (1974a)	3(90)
Luce and Fishburn (1991)	4(82)
Dyer et al. (1992)	5(74)
Fishburn (1982a)	6(67)
Fishburn and Rubinstein (1982)	6(67)
Fishburn (1991)	8(63)
Fishburn and Kochenberger (1979)	9(58)
Fishburn (1977a)	9(58)
Fishburn (1965a)	11(44)
Fishburn (1970b)	12(40)
Fishburn (1967)	13(39)
Brams and Fishburn (1978)	14(38)
Fishburn (1973b)	15(36)

Endnotes

[1] A cleaned copy of the collected data accompanies this article in an online appendix.

[2] All subsequent citation counts in this paper are also as of August 2022 unless stated otherwise.

[3] The 15 most frequently cited journal articles in a set of 19 related journals are provided in the appendix.

[4] As of the writing of this paper, approval voting is specified in the bylaws for INFORMS, available at https://www.informs.org/About-INFORMS/Governance, and in the bylaws for SJDM, available at http://www.sjdm.org/bylaws.pdf.

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