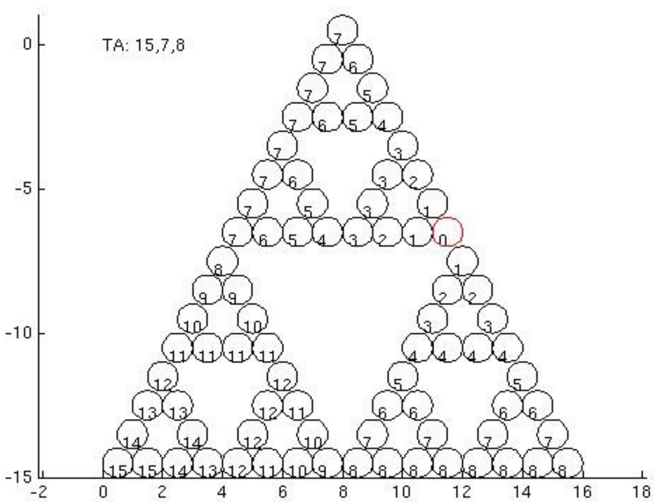
Research Statement Donna Dietz October 2011

Summary

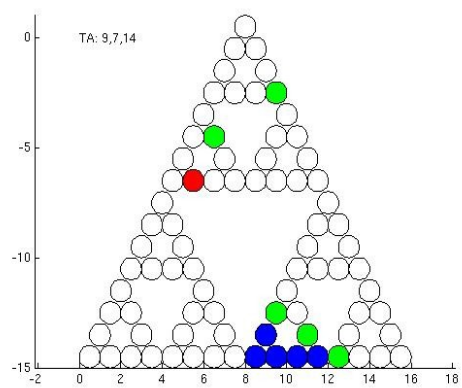
At the age of 11, I discovered that I loved solving puzzles and other problems in mathematics, and by the age of 13, I realized that I had a strong aptitude in this area. I very much appreciate and enjoy problems with applications, but I tend to be most drawn to problems whose statements are more accessible to the undergraduate population.

Towers of Hanoi

For example, I enjoy working on some still-open questions involving the Towers of Hanoi puzzle. The valid positions of a Towers of Hanoi puzzle can be placed in a graph which turns out to be a Sierpinski triangle! Graph adjacencies represent states having a distance of one move. It is surprisingly simple (and fast) to compute the distance between any two valid positions of a Towers of Hanoi. However, this calculation involves computing two possible scenarios. Either the largest disc is moved exactly once (this is the most common arrangment), or the largest disc is moved twice[1]. Additonal progress on this open problem was made by Dan Romik[2] in roughly 2003.



Distances from a given location within a Sierpinski triangle



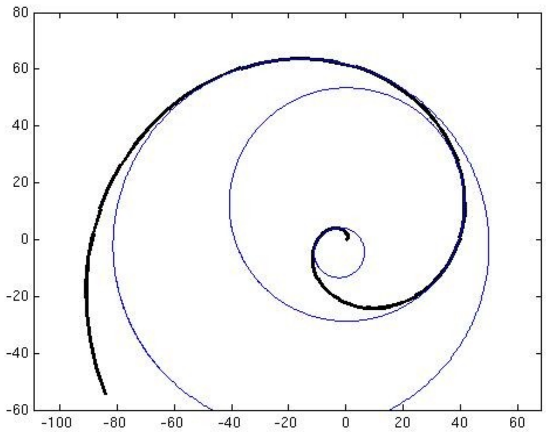
Green points have two minimal paths to the red point. Blue points have the property that the minimal path to the red point requires moving the largest disc twice in the corresponding Hanoi problem.

This semester, I am introducing the simpler aspects of this problem in two of my courses. I am hopeful that after this introduction, I will find some students who would like to participate in research on this or other open problems posed in these areas.

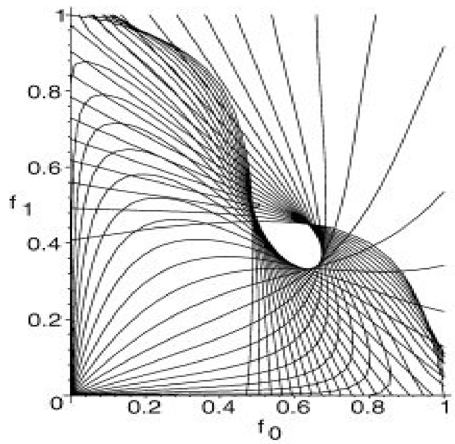
1 This occurs one every 14 times provided the largest disc is required to move at all. (Andreas Hinz, 1992, *Shortest paths between regular states of the Towers of Hanoi*, European Journal of Combinatorics.)
2 Dan Romik, *Shortest Paths in the Tower of Hanoi Graph and Finite Automata*, easily located online, but not dated.

Convex Cubic Spirals

I chose to study mathematics in college and then in graduate school, and I ended up doing research in Computer-Aided Geometric Design. My thesis topic involved using the degrees of freedom present in a planar parametric cubic[3] to ensure monotonic curvature[4] of the cubic. My advisor and I co-authored two papers on the subject, where the second paper expanded the discussion to include rational cubics[5].

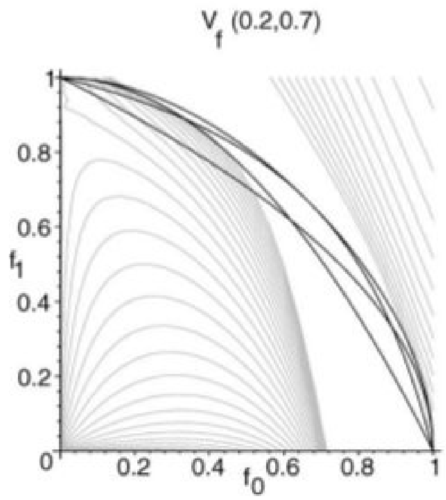


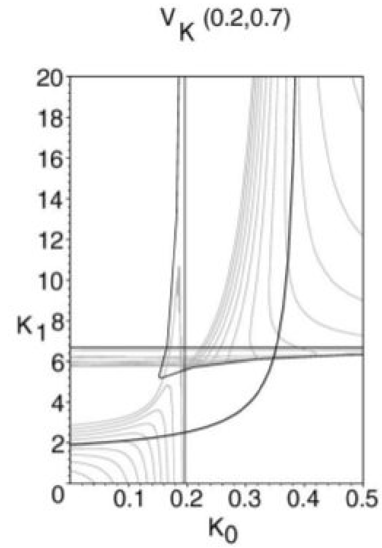
Three nested circles of curvature on a (non cubic) spiral. Note that the spiral is always crossing into its circles of curvature as it is followed inward.



The black curves, found numerically, eliminate possible f0-f1 pairs which can yield cubic spirals. Remining white area indicates cubic spirals. The free variables f0-f1 are explained in the papers.

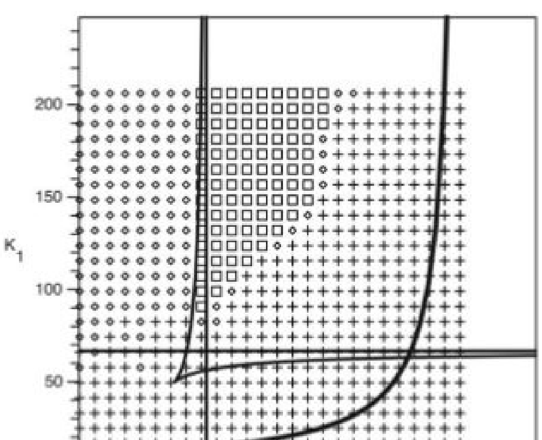
3 That is, a parametric function which has a cubic in *t* for each coordinate.
4 Monotonic curvature is what we mean when we say "spiral" in this context.
5 Rational cubics are composed of a cubic function divided by another cubic function.





The gray curves, found numerically, eliminate possible f0-f1 pairs which can yield cubic spirals. Remining white area indicates cubic spirals. The black curves are for analytical purposes and are found algebraically. Tangents at ends are fixed.

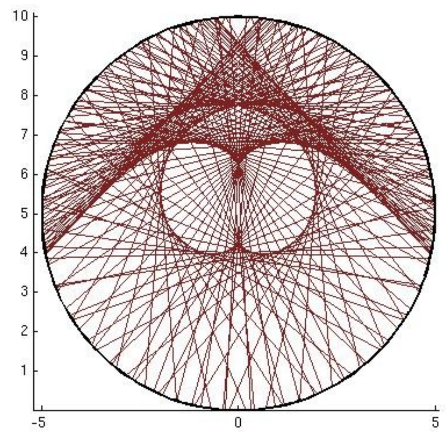
The gray and black curves in this diagram play exactly the same role as in the diagram to its left, except the two free variables are the curvatues at the endpoints of the cubic. The tangent angles at the endpoints are fixed (0.2 and 0.7).



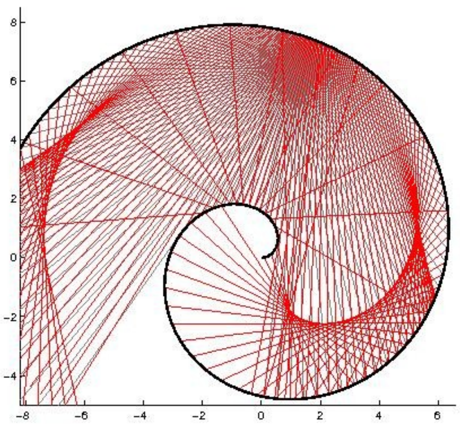
In this diagram (left), rational cubics are numerically analysed for fixed tangential conditions. Squares represent curvature constraints which may be satisfied by cubics alone. Diamonds indicate additional curvature combinations gained by allowing rational cubics rather than just cubics. The curve which begins roughly at (0,10) in the diagram represents the the boundary beyond which no spiral may occur due to simple geometric constraints.

Combinatorial Geometry of Ray-Launching

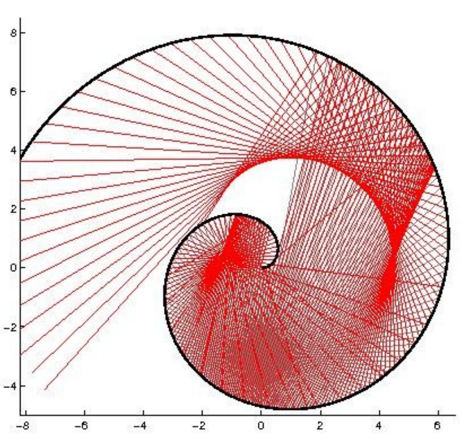
Since arriving at Penn, I've been working on ray-launching problems which are tangentially related to work my husband/colleague is doing in topological imaging. I have been creating geometric simulations of wavefronts in MATLAB with the ultimate goal of making some geometric or topological hypotheses based on what is observed in the geometric simulations. This has been quite interesting and has opened up more questions than it has resolved.



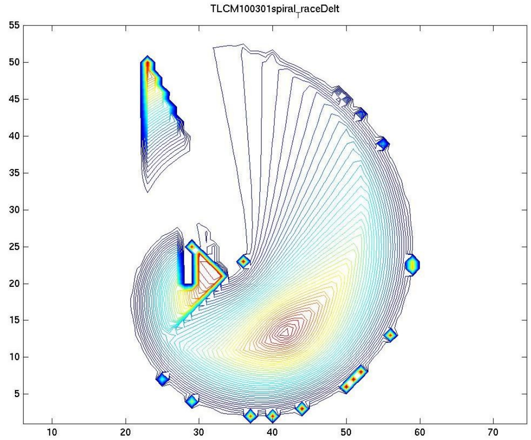
This diagram shows the third level bounces of rays in a circle. The original point of the rays' origin was off-center. Note the lovely heart-shaped caustics!



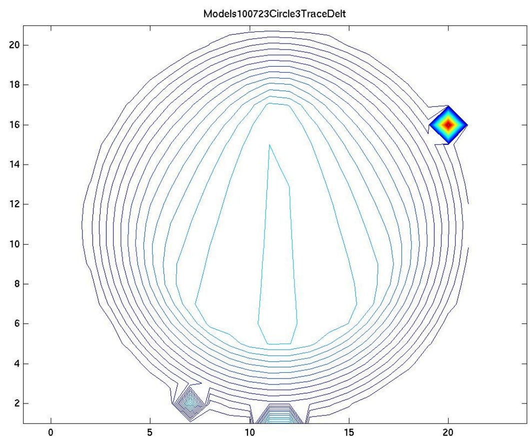
This diagram shows the original set of rays emanating from the source (gray) and the first set of bounced rays. The source is roughly at (2,6) in the diagram. Several characteristic trends are seen. The ever-present caustics are visible, as is the well-hidden center of the spiral. There is a region centered near (3,0), where the bounced rays are nearly parallel, indicating that rays bounced off a region nearly parabaloic in naure.



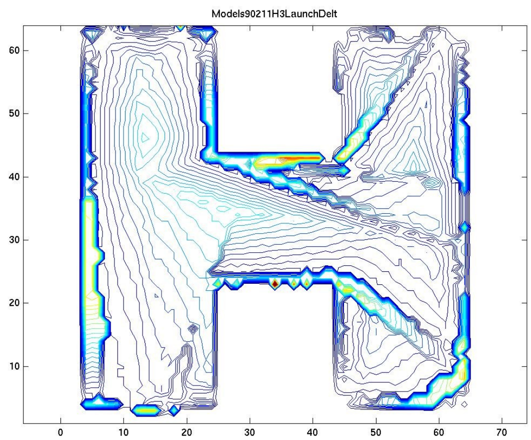
This diagram is the same as the one above, except that the source has moved to a location near (0,-3). The center of the spiral can now "hear" rays sooner.



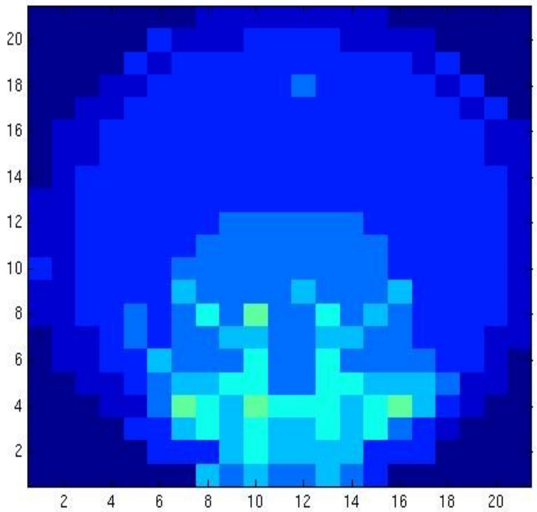
This diagram shows level curves in the time-difference-of-arrival domain (TDOA). This diagram was created using ray-tracing rather than ray-launching, so it is more accurate. However, ray-launching becomes computationally difficult after the first set of bounces, so I use ray-tracing to simulate bounces beyond the first.



Ray-tracing is used again to detect level curves in the TDOA domain. Numerical issues are wildly obvious. However, most of the level curves are fairly smooth, which is what we expect. The level curves near boundaries have small TDOAs while the curves near the middles have larger TDOAs. The hope is to harness this trend to estimate the distance of a sensor to a wall. However, in more complex geometries, the issues compound rapidly.



In this classical H-shaped geometry, ray-launching is used, because many areas of the geometry (which can be thought of as being in naturally occurring "cells") do not hear the first or even second bounce of the rays. Numerics are again quite obvious.



This diagram, which is one frame from an animation, shows which sensors have heard the signal a given number of times. The geometry is a simple circle with an off-center source.

The Future

While I wish to continue the ray-launching research, my heart is still drawn most strongly to problems involving very simplistic games or other scenarios. Ultimately, I want to become better acquainted with open research questions in those (or similar) areas which I can work on with interested undergraduate students.

Research progress for me has been slow but steady as my time has been limited. I accepted a position in a teaching-oriented university (Mansfield University) after graduate school, and I needed to participate in many non-research related activities there in order to work towards tenure. I was granted tenure and promotion at Mansfield University, and then left for a one-year leave of absence and took a position as Visiting Scholar and then Lecturer in the Mathematics department at the University of Pennsylvania. I resigned from Mansfield University due to my two-body problem and accepted a position as Lecturer in the Computer & Information Sciences department at the Univeristy of Pennsylvania. Each time I switch departments, this takes yet more time away from my research budget, as I have always put course preparation ahead of my own research. Also, I've acquired a new field insofar as teaching is concerned, and doing this well has been a high priority for me. This is why the conferences I've attended have revolved around teaching rather than resarch.

For each of the three research areas described above, more details, my articles, and diagrams are available on my website at:

http://www.cis.upenn.edu/~dietzd/research