Mathematical Appendix to
Open Economy Models of Distribution and Growth*

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Steady State Values of Endogenous Variables in the Medium-Run Model of Income Distrtibution and the Real Exchange Rate

In order to determine the steady state values of the wage share, ψ and real exchange rate, q, we begin with equations (17′) and (18′) from the text:

$$\hat{\psi} = \phi(\psi_w - \psi) + \gamma q - \epsilon - \theta(\psi - \tau + \beta q) \tag{17′}$$

$$\hat{q} = \mu(\bar{q} - q) + p^* - \theta(\psi - \tau + \beta q) \tag{18′}$$

which can be rewritten as:

$$\hat{\psi} = -(\phi + \theta)\psi + (\gamma - \theta\beta)q + \phi\psi_w + \theta\tau - \epsilon$$

$$\hat{q} = -\theta\psi - (\mu + \theta\beta)q + \mu\bar{q} + p^* + \theta\tau$$

Setting $\hat{\psi} = 0$ and $\hat{q} = 0$, we can solve for the steady values of ψ and q writing (17′) and (18′) in matrix form:

$$\begin{bmatrix} -(\phi+\theta) & \gamma - \theta\beta \\ -\theta & -(\mu + \theta\beta) \end{bmatrix} \begin{bmatrix} \psi \\ q \end{bmatrix} = \begin{bmatrix} -\phi\psi_w - \theta\tau + \epsilon \\ -\mu\bar{q} - p^* - \theta\tau \end{bmatrix} \tag{A.1}$$

Define the Jacobian $\mathbf{J}$ as:

$$\mathbf{J} = \begin{bmatrix} -(\phi+\theta) & \gamma - \theta\beta \\ -\theta & -(\mu + \theta\beta) \end{bmatrix}$$

The stability conditions are definitely satisfied since:

$$Tr(\mathbf{J}) = -(\phi + \theta) - (\mu + \theta\beta) < 0$$

$$|\mathbf{J}| = \mu(\phi + \theta) + \theta(\phi\beta + \gamma) > 0$$

Using this matrix notation, we can solve for the slopes of the FE and DC curves:

$$\left.\frac{\partial\psi}{\partial q}\right|_{FE} = \frac{-(\mu + \theta\beta)}{\theta} < 0$$

$$\left.\frac{\partial\psi}{\partial q}\right|_{DC} = \frac{\gamma - \theta\beta}{\theta + \phi}$$

*Thanks to Barton Baker for assistance in the preparation of this appendix.

When both curves slope down,

Note that $|\mathbf{J}| > 0$ is equivalent to FE being steeper (i.e., more negatively sloped). Solving and simplifying from A.1 for ψ and q, we find the following reduced-form solutions:

$$\psi = \frac{\mu(\phi\psi_w + \theta\tau - \epsilon) + \theta\beta(\phi\psi_w - \epsilon - \mu\bar{q} - p^*) + \gamma(\mu\bar{q} + p^* + \theta\tau)}{\mu(\phi + \theta) + \theta(\phi\beta + \gamma)} \tag{A.2}$$

$$q = \frac{\theta(-\phi\psi_w - \theta\tau + \epsilon) + (\phi + \theta)(\mu\bar{q} + p^* + \theta\tau)}{\mu(\theta + \phi) + \theta(\phi\beta + \gamma)} \tag{A.3}$$

To solve for the medium run equilibrium inflation rate, we start with (15′):

$$\hat{P} = \theta(\psi - \tau + \beta q)$$

Plugging in (A.2) and (A.3) we get:

$$\hat{P} = \frac{\theta[\mu(\phi\psi_w - \epsilon) - \mu\phi\tau + (\gamma + \phi\beta)(\mu\bar{q} + p^*)]}{\mu(\phi + \theta) + \theta(\phi\beta + \gamma)}$$

Comparative Statics

We can use these steady state solutions to perform some comparative statics with regard to the effects of shifts in various parameters on the medium-run equilibrium:

The effect of an increase in worker's target wage-share on the steady-state wage share is definitely positive:

$$\frac{d\psi}{d\psi_w} = \frac{\phi(\mu + \theta\beta)}{|\mathbf{J}|} > 0 \tag{A.4}$$

The effect of an increase in target wage-share on the steady-state real exchange rate is definitely negative:

$$\frac{dq}{d\psi_w} = \frac{-\phi\theta}{|\mathbf{J}|} < 0 \tag{A.5}$$

The effect of an increase in the market power parameter (inversely related to market power) on the steady-state wage share is definitely positive:

$$\frac{d\psi}{d\tau} = \frac{\theta(\mu + \gamma)}{|\mathbf{J}|} > 0 \tag{A.6}$$

The effect of an increase in the market power parameter (inversely related to market power) on the steady-state real exchange rate is positive (i.e., causes a real depreciation):

$$\frac{dq}{d\tau} = \frac{\theta\phi}{|\mathbf{J}|} > 0 \tag{A.7}$$

The effect of an increase in the medium-run real exchange rate target on the steady-state wage share is given by:

$$\frac{d\psi}{d\bar{q}} = \frac{\mu(\gamma - \theta\beta)}{|\mathbf{J}|} \tag{A.8}$$

which is positive if DC slopes upward and negative if DC slopes downward.

The effect of an increase in the medium-run real exchange rate target on the steady-state real exchange rate is necessarily positive:

$$\frac{dq}{d\bar{q}} = \frac{(\phi + \theta)\mu}{|\mathbf{J}|} > 0 \tag{A.9}$$

Finally, the effect of an increase in the medium-run real exchange rate target on steady-state inflation is also necessarily positive:

$$\frac{d\hat{P}}{d\bar{q}} = \frac{\mu\theta(\gamma + \beta\phi)}{|\mathbf{J}|} > 0 \tag{A.10}$$