Mathematical Appendix to
Open Economy Models of Distribution and Growth∗

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Steady State Values of Endogenous Variables in the Medium-Run Model of Income Distribution and the Real Exchange Rate

In order to determine the steady state values of the wage share, ψ and real exchange rate, q, we begin with equations (17′) and (18′) from the text:

\[ \dot{\psi} = \phi(\psi_w - \psi) + \gamma q - \epsilon - \theta(\psi - \tau + \beta q) \] (17′)
\[ \dot{q} = \mu(\bar{q} - q) + p^* - \theta(\psi - \tau + \beta q) \] (18′)

which can be rewritten as:

\[ \dot{\psi} = -(\phi + \theta)\psi + (\gamma - \theta \beta)q + \phi \psi_w + \theta \tau - \epsilon \]
\[ \dot{q} = -\theta \psi - (\mu + \theta \beta)q + \mu \bar{q} + p^* + \theta \tau \]

Setting \( \dot{\psi} = 0 \) and \( \dot{q} = 0 \), we can solve for the steady values of \( \psi \) and \( q \) writing (17′) and (18′) in matrix form:

\[ \begin{bmatrix} -\phi - \theta \\ -\theta \end{bmatrix} \begin{bmatrix} \psi \\ q \end{bmatrix} = \begin{bmatrix} -\phi \psi_w - \theta \tau + \epsilon \\ -\mu \bar{q} - p^* - \theta \tau \end{bmatrix} \] (A.1)

Define the Jacobian \( J \) as:

\[ J = \begin{bmatrix} -\phi - \theta & \gamma - \theta \beta \\ -\theta & -\mu - \theta \beta \end{bmatrix} \]

The stability conditions are definitely satisfied since:

\[ \text{Tr}(J) = -\phi - \theta - (\mu + \theta \beta) < 0 \]
\[ |J| = \mu(\phi + \theta) + \theta(\phi \beta + \gamma) > 0 \]

Using this matrix notation, we can solve for the slopes of the FE and DC curves:

\[ \frac{\partial \psi}{\partial q}\bigg|_{\text{FE}} = \frac{-(\mu + \theta \beta)}{\theta} < 0 \]
\[ \frac{\partial \psi}{\partial q}\bigg|_{\text{DC}} = \frac{\gamma - \theta \beta}{\theta + \phi} \]

∗Thanks to Barton Baker for assistance in the preparation of this appendix.
When both curves slope down,

Note that $|J| > 0$ is equivalent to FE being steeper (i.e., more negatively sloped).
Solving and simplifying from A.1 for $\psi$ and $q$, we find the following reduced-form solutions:

$$\psi = \frac{\mu(\phi \psi_w + \theta \tau - \epsilon) + \theta \beta (\phi \psi_w - \epsilon - \mu \bar{q} - p^*) + \gamma (\mu \bar{q} + p^* + \theta \tau)}{\mu(\phi + \theta) + \theta (\phi \beta + \gamma)}$$ (A.2)

$$q = \frac{\theta (\phi \psi_w - \theta \tau + \epsilon) + (\phi + \theta) (\mu \bar{q} + p^* + \theta \tau)}{\mu(\theta + \phi) + \theta (\phi \beta + \gamma)}$$ (A.3)

To solve for the medium run equilibrium inflation rate, we start with (15′):

$$\hat{P} = \theta (\psi - \tau + \beta q)$$

Plugging in (A.2) and (A.3) we get:

$$\hat{P} = \theta \left[ \mu (\phi \psi_w - \theta \tau + \epsilon) + (\phi + \theta) (\mu \bar{q} + p^* + \theta \tau) \right]$$

$$\mu(\phi + \theta) + \theta (\phi \beta + \gamma)$$

**Comparative Statics**

We can use these steady state solutions to perform some comparative statics with regard to the effects of shifts in various parameters on the medium-run equilibrium:

The effect of an increase in worker’s target wage-share on the steady-state wage share is definitely positive:

$$\frac{d\psi}{d\psi_w} = \frac{\phi(\mu + \theta \beta)}{|J|} > 0$$ (A.4)

The effect of an increase in target wage-share on the steady-state real exchange rate is definitely negative:

$$\frac{dq}{d\psi_w} = -\frac{\phi \theta}{|J|} < 0$$ (A.5)

The effect of an increase in the market power parameter (inversely related to market power) on the steady-state wage share is definitely positive:

$$\frac{d\psi}{d\tau} = \frac{\theta(\mu + \gamma)}{|J|} > 0$$ (A.6)

The effect of an increase in the market power parameter (inversely related to market power) on the steady-state real exchange rate is positive (i.e., causes a real depreciation):

$$\frac{dq}{d\tau} = \frac{\theta \phi}{|J|} > 0$$ (A.7)

The effect of an increase in the medium-run real exchange rate target on the steady-state wage share is given by:

$$\frac{d\psi}{d\bar{q}} = \frac{\mu(\gamma - \theta \beta)}{|J|}$$ (A.8)
which is positive if DC slopes upward and negative if DC slopes downward.

The effect of an increase in the medium-run real exchange rate target on the steady-state real exchange rate is necessarily positive:

$$\frac{dq}{dq} = \frac{(\phi + \theta)\mu}{|J|} > 0$$  \hspace{1cm} (A.9)

Finally, the effect of an increase in the medium-run real exchange rate target on steady-state inflation is also necessarily positive:

$$\frac{d\hat{P}}{dq} = \frac{\mu\theta(\gamma + \beta\phi)}{|J|} > 0$$  \hspace{1cm} (A.10)