Chi-Square Tests

Learning Objectives

In this chapter, you learn:

- How and when to use the chi-square test for contingency tables
- How to test multiple proportions
- How to test for independence

Contingency Table Example

Left-Handed vs. Gender

Dominant Hand: Left vs. Right

Gender: Male vs. Female

- 2 categories for each variable, so this is called a 2 x 2 table
- Suppose we examine a sample of 300 children

Contingency Table Example

(continued)

Sample results organized in a **contingency table**:

sample size = n = 300:

120 Females, 12 were left handed

180 Males, 24 were left handed

Gender	Hand Preference		
	Left	Right	
Female	12	108	120
Male	24	156	180
	36	264	300

Test for the Equality Between Proportions

H_0: $\pi_1 = \pi_2$ (Proportion of females who are left handed is equal to the proportion of males who are left handed)

H_1: $\pi_1 \neq \pi_2$ (The two proportions are not the same – hand preference is not independent of gender)

- If H_0 is true, then the proportion of left-handed females should be the same as the proportion of left-handed males. Left-handedness is then *independent* of gender.

General Problem – Test of Independence

H_0: The two categorical variables are independent
(i.e., there is no relationship between them)

H_1: The two categorical variables are dependent
(i.e., there is a relationship between them)

Each variable may have more than two categories, extending to contingency tables with r rows and c columns

The Chi-Square Test Statistic

The Chi-square (χ^2) test statistic is:

$$\chi^2_{STAT} = \sum_{all\,\text{cells}} \frac{(f_o - f_e)^2}{f_e}$$

- where:

  f_o = observed frequency in a particular cell of the r x c table

  f_e = expected frequency in a particular cell if H_0 is true (next slide)

χ^2_{STAT} for the r x c table has (r - 1)(c - 1) degrees of freedom

(Assumed: each cell in the contingency table has expected frequency of at least 1. Otherwise, merge some categories.)

Expected Cell Frequencies

- Expected cell frequencies:

$$f_e = \frac{\text{row total} \times \text{column total}}{n}$$

Where:

row total = sum of all frequencies in the row

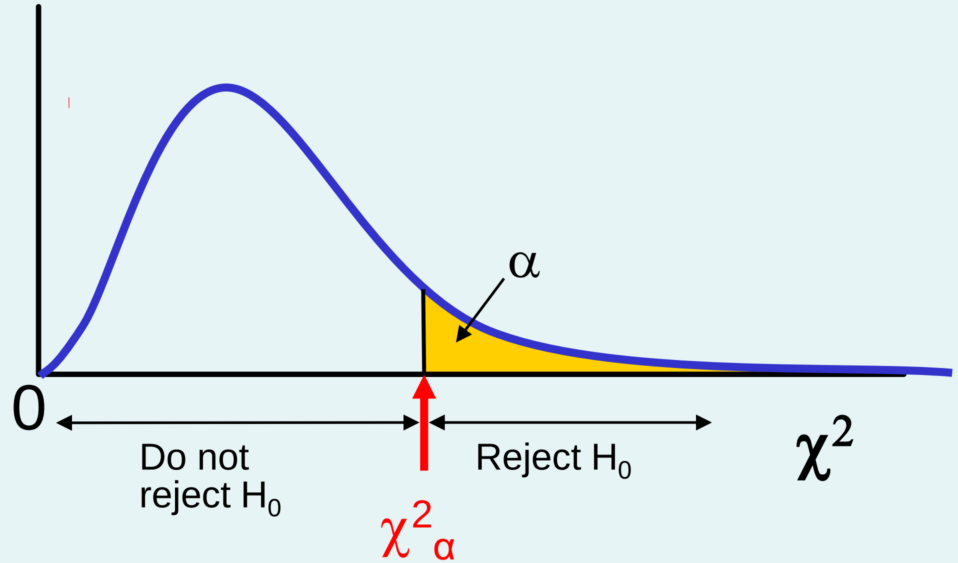
column total = sum of all frequencies in the column

n = overall sample size

Decision Rule

The χ^2_{STAT} test statistic approximately follows a chi-squared distribution with (r – 1)(c – 1) degrees of freedom

Decision Rule:
If $\chi^2_{STAT} > \chi^2_{\alpha}$, reject H_0, otherwise, do not reject H_0



Special case: two proportions

Example: Gender vs Lefthandedness

Gender	Hand Preference		
	Left	Right	
Female	Observed = 12 Expected = 14.4	Observed = 108 Expected = 105.6	120
Male	Observed = 24 Expected = 21.6	Observed = 156 Expected = 158.4	180
	36	264	300

The Chi-Square Test Statistic

Gender	Hand Preference: Left	Hand Preference: Right	
Female	Observed = 12 Expected = 14.4	Observed = 108 Expected = 105.6	120
Male	Observed = 24 Expected = 21.6	Observed = 156 Expected = 158.4	180
	36	264	300

The test statistic is:

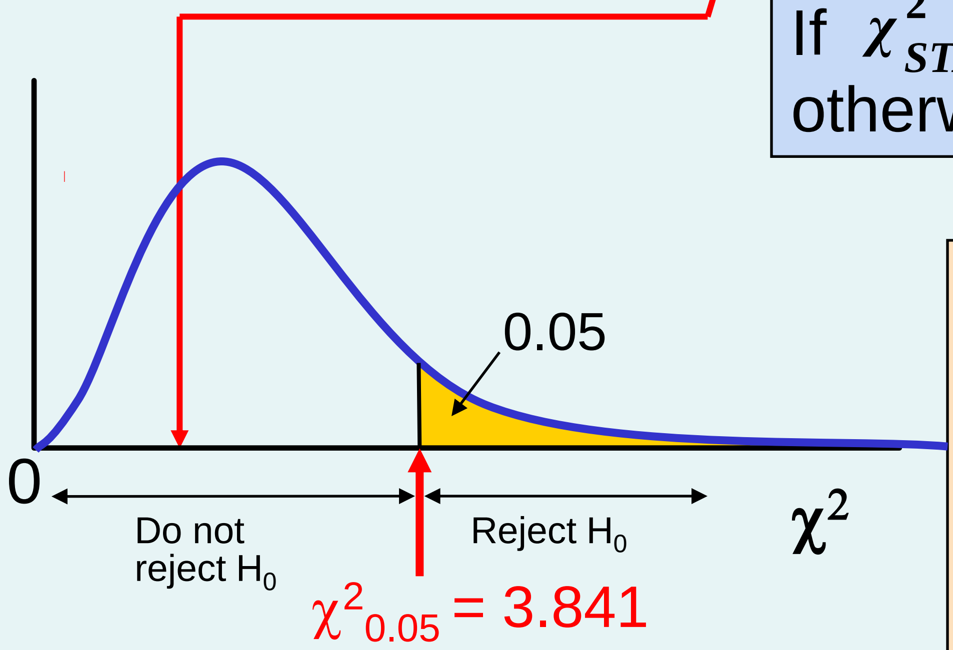
$$\chi^2_{STAT} = \sum_{\text{all cells}} \frac{(f_o - f_e)^2}{f_e}$$

$$= \frac{(12-14.4)^2}{14.4} + \frac{(108-105.6)^2}{105.6} + \frac{(24-21.6)^2}{21.6} + \frac{(156-158.4)^2}{158.4} = 0.7576$$

Decision Rule

The test statistic is $\chi^2_{STAT} = 0.7576$; $\chi^2_{0.05}$ with 1 d.f. $= 3.841$

Decision Rule:
If $\chi^2_{STAT} > 3.841$, reject H_0, otherwise, do not reject H_0



Here,
$\chi^2_{STAT} = 0.7576 < \chi^2_{0.05} = 3.841$, so we do not reject H_0 and conclude that there is not sufficient evidence that the two proportions are different at $\alpha = 0.05$

Special case: more than two proportions

- More than two independent populations

Test

H_0: $\pi_1 = \pi_2 = \cdots = \pi_c$

H_1: Not all of the π_j are equal ($j = 1, 2, \cdots, c$)

$$\chi^2_{STAT} \text{ for the 2 x c case has } (2-1)(c-1) = c-1 \text{ degrees of freedom}$$

Example: c = 3 categories

$H_0: \pi_1 = \pi_2 = \pi_3$

H_1: Not all of the π_j are equal (j = 1, 2, 3)

Chi-Square Test: Administrators, Students, Faculty

	Admin	Students	Faculty	Total
Favor	63	20	37	120
	60	30	30	
Oppose	37	30	13	80
	40	20	20	
Total	100	50	50	200

Expected (rows 60, 30, 30 and 40, 20, 20)

Observed (rows 63, 20, 37 and 37, 30, 13)

$$\chi^2_{STAT} = 12.792 > \chi^2_{0.01} = 9.2103, \text{ so reject } H_0$$

General case of $r \times c$ table

- Example: the meal plan selected by 200 students

Class Standing	Number of meals per week			Total
	20/week	10/week	none	
Fresh.	24	32	14	70
Soph.	22	26	12	60
Junior	10	14	6	30
Senior	14	16	10	40
Total	70	88	42	200

Null and alternative hypotheses

- The hypothesis to be tested is:

H_0: Meal plan and class standing are independent
(i.e., there is no relationship between them)
H_1: Meal plan and class standing are dependent
(i.e., there is a relationship between them)

Expected Cell Frequencies

Observed:

Class Standing	Number of meals per week			Total
	20/wk	10/wk	none	
Fresh.	24	32	14	70
Soph.	22	26	12	60
Junior	10	14	6	30
Senior	14	16	10	40
Total	70	88	42	200

Expected cell frequencies if H_0 is true:

Class Standing	Number of meals per week			Total
	20/wk	10/wk	none	
Fresh.	24.5	30.8	14.7	70
Soph.	21.0	26.4	12.6	60
Junior	10.5	13.2	6.3	30
Senior	14.0	17.6	8.4	40
Total	70	88	42	200

Example for one cell:

$$f_e = \frac{\text{row total} \times \text{column total}}{n}$$

$$= \frac{30 \times 70}{200} = 10.5$$

The Test Statistic

- The test statistic value is:

$$\chi^2_{STAT} = \sum_{all\ \text{cells}} \frac{(f_o - f_e)^2}{f_e}$$

$$= \frac{(24 - 24.5)^2}{24.5} + \frac{(32 - 30.8)^2}{30.8} + \cdots + \frac{(10 - 8.4)^2}{8.4} = 0.709$$

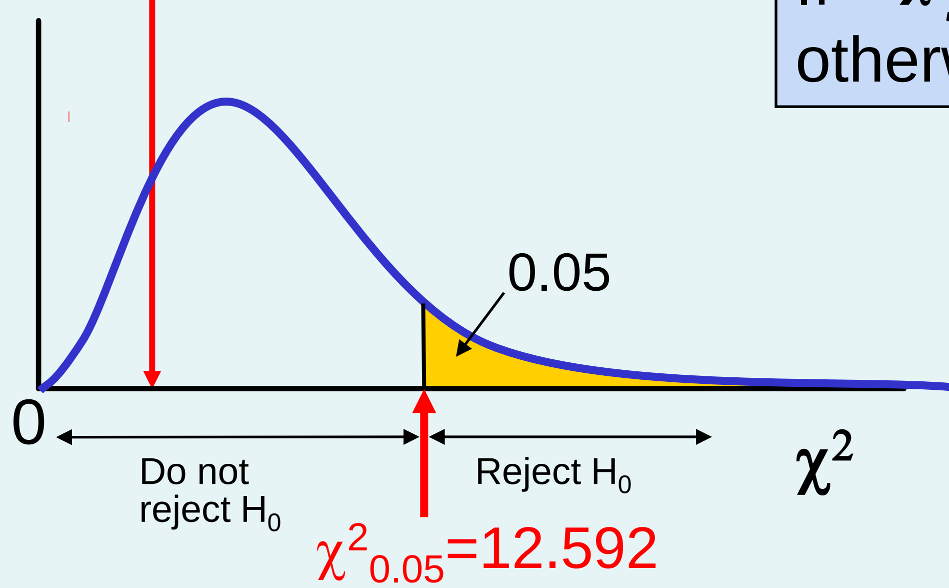
$\chi^2_{0.05}$ = 12.592 from the chi-squared distribution with (4 – 1)(3 – 1) = 6 degrees of freedom

Decision and Interpretation

(continued)

The test statistic is $\chi^2_{STAT} = 0.709$; $\chi^2_{0.05}$ with 6 d.f. $= 12.592$

Decision Rule:
If $\chi^2_{STAT} > 12.592$, reject H_0, otherwise, do not reject H_0



Here,
$\chi^2_{STAT} = 0.709 < \chi^2_{0.05} = 12.592$,
so do not reject H_0

Conclusion: there is not sufficient evidence that meal plan and class standing are related at $\alpha = 0.05$

Chapter Summary

In this chapter we discussed

- Applying the χ^2 test for the difference between two proportions
- Applying the χ^2 test for differences in more than two proportions
- Applying the χ^2 test for independence