1. Let $\mathcal{M}$ and $\mathcal{N}$ be differentiable manifolds of dimensions, respectively, $m$ and $n$. Prove that $\mathcal{M} \times \mathcal{N}$ is a differentiable manifold of dimension $m + n$. It follows that the torus $S^1 \times S^1$ is a differentiable two dimensional manifold.

2. Prove that there is a bijection between the set of differentiable functions on $S^1$ and the set
\[ \{ f : \mathbb{R}^1 \to \mathbb{R}^1 | f \text{ is differentiable and periodic with period } 2\pi \} \]

3. Prove that, while of the same dimension, $S^2$ and $S^1 \times S^1$ are different manifolds, i.e. there is no diffeomorphism $\phi : S^1 \times S^1 \to S^2$. 
